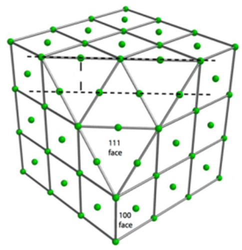
Tentamen i

Vågfysik

för FyN (NFYB01), Y/Yi (TFYA10) och MED (TFYA59)

Lösningar till tentamen **2013-01-11**



#18

1.

$$f_m = \frac{v - v_m}{v - v_s} \cdot f_0$$

m mottagare (obs)

s sändare

a) $m \xleftarrow{+} S$ Linnea

$$f_{Linnea} = \frac{330 + 10}{330 + 10} \cdot 440 = \underline{\underline{440 \text{ Hz}}}$$

$S \xrightarrow{+} m$ Linus

$$f_{Linus} = \frac{330 + 10}{330 - 10} = \underline{\underline{467,5 \text{ Hz}}}$$

b)

$$f_{Linnea} = \frac{330 + 10}{330 - 0} = \underline{\underline{453,3 \text{ Hz}}}$$

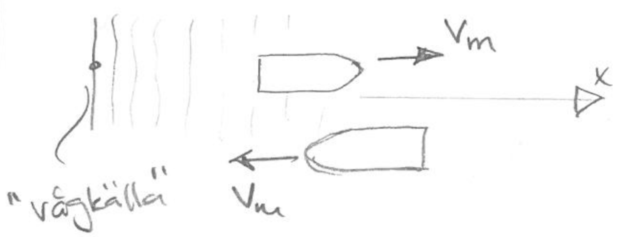
$$f_{Linus} = \frac{330 + 10}{330 - 0} = \underline{\underline{453,3 \text{ Hz}}}$$

c)

$$f_{Linnea} = \frac{330 - 0}{330 + 10} = \underline{\underline{427,0 \text{ Hz}}}$$

$$f_{Linus} = \frac{330 - 0}{330 - 10} = \underline{\underline{453,75 \text{ Hz}}}$$

4.



$V_m = 3\ km/h = 0{,}8333\ldots\ m/s$

Uppleud frekvens:

$+x \quad f_{m+} = 0{,}2$ vågor/s

$-x \quad f_{m-} = 0.5$ vågor/s

Vågkällan kan antas vara i vila $V_S = 0$

d) $f_m = f_s \dfrac{V \mp V_m}{V}$

$$f_{m+} = f_S \frac{V - V_m}{V} \quad (1)$$

$$f_{m-} = f_S \frac{V + V_m}{V} \quad (2)$$

(1): $f_S = \underbrace{f_{m+} \dfrac{V}{V - V_m}}$

(2): $f_{m-} = f_S \dfrac{V+V_m}{V} = f_{m+} \dfrac{\cancel{V}}{V - V_m} \cdot \dfrac{V+V_m}{\cancel{V}} \Rightarrow$

$\Rightarrow f_{m-} \cdot V - f_{m-} \cdot V_m = f_{m+} \cdot V + f_{m+} \cdot V_m \Rightarrow$

$\Rightarrow V = \dfrac{(f_{m-} + f_{m+}) \cdot V_m}{(f_{m-} - f_{m+})} \quad \therefore V = \dfrac{0{,}5 + 0{,}2}{0{,}5 - 0{,}2} \cdot 3 = 7\ km/h =$

$= 1{,}9444\ldots\ m/s$

$$\underline{\underline{V = 7\ km/h}} \simeq 1{,}9\ m/s$$

e) (1): $f_S = 0{,}2 \dfrac{7}{7-3} = 0{,}35\ s^{-1}$

$\lambda = \dfrac{V}{f_S} = \dfrac{7/3{,}6}{0{,}35} = 5{,}555\ldots\ m$

$\lambda = 5{,}6\ m$

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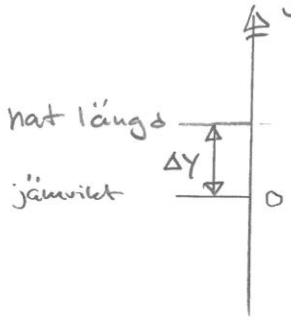
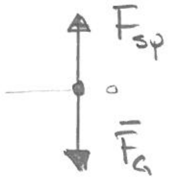
2.

a)



Vid jämvikt:



$$\bar{F}_G + \bar{F}_{sp} = 0 \Rightarrow$$

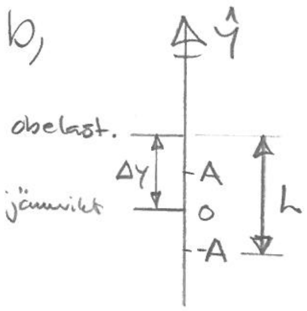
$$\Rightarrow -mg\hat{y} + k \cdot \Delta y\,\hat{y} = 0 \Rightarrow$$

$$\Rightarrow mg = k \cdot \Delta y$$

Godt. läge: $\bar{F}_{net} = \bar{F}_G + \bar{F}_{sp} = -mg\hat{y} + k(\Delta y - y)\hat{y} \Rightarrow$

$$\left.\begin{aligned} \bar{F}_{net} &= -mg\hat{y} + mg\hat{y} - ky\hat{y} = -ky\hat{y} \\ \bar{F}_{net} &= m\ddot{y} \end{aligned}\right\} \underline{\underline{\ddot{y} + \frac{k}{m}y = 0}}$$

b)



Jämvikt: $\bar{F}_G + \bar{F}_{sp} = 0 \Rightarrow -mg\hat{y} + k\Delta y\,\hat{y} = 0 \Rightarrow$

$$\Rightarrow mg = k \cdot \Delta y \Rightarrow \Delta y = \frac{mg}{k}$$

Harm. rörelse beskrivs av

$$y(t) = A \cdot \cos(\omega t + \phi)$$

där $A = L - \Delta y$ och $\omega = \sqrt{\frac{k}{m}}$

Vid $t=0$ $\quad y(0) = A\cos(\phi) = -A$ längst ned

$\therefore \phi = \pi$

med $m = 65$ kg, $k = 270$ N/m, $L = 5$ m och $t = 2$ s:

Radianer!

$$y(2) = \underbrace{\left(5 - \frac{65 \cdot 9.8}{270}\right)}_{2,64\ldots} \cdot \underbrace{\cos\left(\sqrt{\frac{270}{65}} \cdot 2 + \pi\right)}_{+0.594} = 1,568\ldots$$

$\underline{\underline{\text{Svar: } y = 1,6\text{ m}}}$

c) $\dot{y}(t) = -A\omega \cdot \sin(\omega t + \phi)$

$= -4,329$ m/s

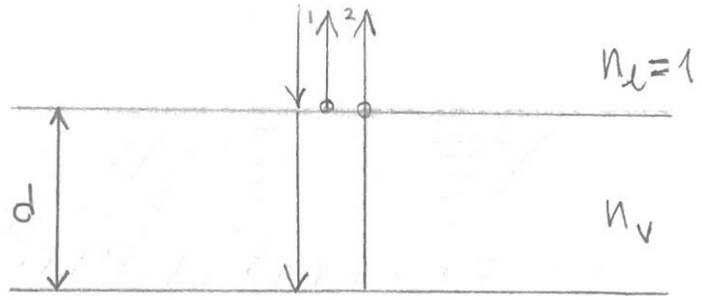
$$\dot{y}(2) = -\left(5 - \frac{65 \cdot 9,8}{270}\right)\sqrt{\frac{270}{65}} \cdot \sin\left(\sqrt{\frac{270}{65}} \cdot 2 + \pi\right) = -4,32\ldots$$

$\underline{\underline{\text{Svar: } \dot{y} = -4,3\text{ m/s}}}$

(18)

3

a)



Betrakta interf.termen $2A_1A_2\cos(\Delta\phi)$ där

fasvinkelskillnaden $\Delta\phi$ beror av ljusets vägskillnad samt de fasskift som uppstår vid reflektionerna.

$$\Delta\phi = \phi_2 - \phi_1 = k_2 x_2 + \phi_{vs} - k_1 x_1 - \phi_{lv}$$

Ref. mot "tätare" mat ($n_s > n_v > n_l$) ger $\phi_{lv} = \phi_{vs} = \pi$

$x_1 = 0$, $x_2 = 2d$

$n_v = \frac{\lambda}{\lambda_v}$

$$\Delta\phi = k_2 \cdot 2d + \pi - 0 - \pi = k_2 \cdot 2d = \frac{2\pi}{\lambda_v} \cdot 2d = \frac{2\pi}{\lambda} \cdot n_v \cdot 2d$$

$$\therefore \Delta\phi = \frac{2\pi}{\lambda} \cdot \underbrace{2dn_v}_{\text{Opt. vägskillnad } \Delta L}$$

Destruktiv interferens $\Delta\phi = (m + \frac{1}{2})2\pi$, $\lambda = \lambda_d$

$$\frac{\cancel{2\pi}}{\lambda_d} \cdot 2dn_v = (m + \tfrac{1}{2})\cancel{2\pi} \Rightarrow 2dn_v = (m + \tfrac{1}{2})\lambda_d$$

Konstruktiv interferens $\Delta\phi = m \cdot 2\pi$, $\lambda = \lambda_k$

$$\frac{2\pi}{\lambda_k} \cdot 2dn_v = m \cdot 2\pi \Rightarrow 2dn_v = m \cdot \lambda_k$$

$$\therefore m = \frac{2dn_v}{\lambda_d} - \frac{1}{2}, \quad m = \frac{2dn_v}{\lambda_k} \Rightarrow \frac{2dn_v}{\lambda_d} - \frac{1}{2} = \frac{2dn_v}{\lambda_k}$$

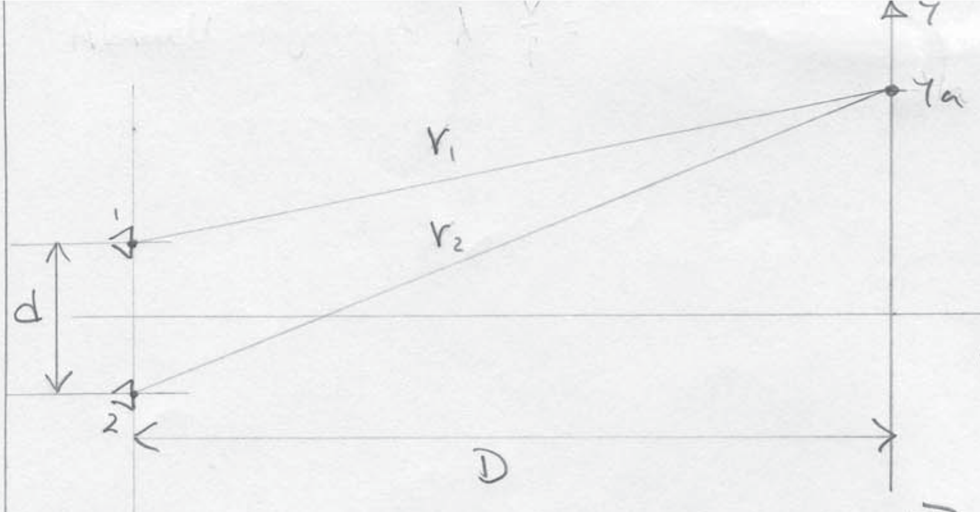
$$\Rightarrow 2dn_v = \frac{1}{2}\left(\frac{1}{\lambda_d} - \frac{1}{\lambda_k}\right)^{-1} \Rightarrow \underline{\underline{d = \frac{1}{4 \cdot n_v}\left(\frac{1}{\lambda_d} - \frac{1}{\lambda_k}\right)^{-1}}}$$

b) $\lambda_d = 590$ nm, $\lambda_k = 740$ nm, $n_v \approx 1{,}35$ ← ungf. samma värde för de två våglängderna

$$d = \frac{1}{4 \cdot 1{,}35}\left(\frac{1}{590} - \frac{1}{740}\right)^{-1} = 539{,}0123\ldots$$

<u>Svar:</u> $\underline{\underline{d = 539 \text{ nm}}}$

4ab)



$d = 2{,}0$ m
$D = 10{,}0$ m
$y_a = 6{,}0$ m

$v = 340$ m/s
$f = 850$ Hz
$\Rightarrow \lambda = \frac{v}{f} = 0{,}4$ m

$r_2 - r_1 = m\lambda$ ger maxima $(m \in \mathbb{Z})$

a) Vid punkt y_a är $r_1 = \sqrt{D^2 + (y_a - \frac{d}{2})^2} = \sqrt{125}$

och $r_2 = \sqrt{D^2 + (y_a + \frac{d}{2})^2} = \sqrt{149}$

$r_2 - r_1 = \sqrt{149} - \sqrt{125} = 1{,}0262\ldots$ m

Maxima registreras då $r_2 - r_1 = 0$ $(m=0)$, $r_2 - r_1 = 0{,}4$ $(m=1)$ och $r_2 - r_1 = 0{,}8$ $(m=2)$

Dvs. tre stycken

b) Första min ges då $r_2 - r_1 = \frac{\lambda}{2} = 0{,}2$

$r_2 - r_1 = \sqrt{100 + (y+1)^2} - \sqrt{100 + (y-1)^2}$

provar

$y = 2$ ger $r_2 - r_1 = \sqrt{109} - \sqrt{101} = 0{,}390\ldots$

$y = 1$ ger $r_2 - r_1 = \sqrt{104} - \sqrt{100} = 0{,}198\ldots$

$y = 1{,}1$ ger $r_2 - r_1 = 0{,}217\ldots$

$y = 1{,}01$ ger $r_2 - r_1 = 0{,}1999\ldots$

$y = 1{,}02$ ger $r_2 - r_1 = 0{,}201\ldots$

Svar: $y = 1{,}01$ m

4c)

c) Intensiteten avståndet r från en källa kan skrivas $I = \frac{P}{\Delta S}$ där $\Delta S = 4\pi r^2$

$P =$ 4 W

om källorna är rundstrålande.

Den sammanlagda intensiteten vid minimat,

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2} \quad \text{där}$$

$$I_1 = \frac{P}{4\pi r_1^2} = \frac{4}{4\pi(100+(0{,}01)^2)}$$

$$I_2 = \frac{P}{4\pi r_2^2} = \frac{4}{4\pi(100+(2{,}01)^2)}$$

Svar: $I =$ $1{,}2.10^{-6}$ W/m^2

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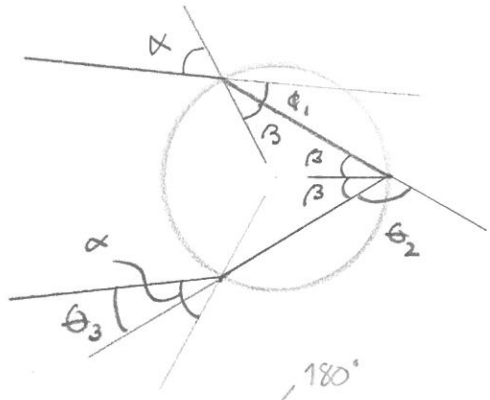
5.

a)

$$\delta(\alpha) = \Phi_1 + \Phi_2 + \Phi_3$$



$$\Phi_1 = \alpha - \beta$$

$$\Phi_2 = \pi - 2\beta$$

$$\Phi_3 = \alpha - \beta$$

180°

$$\therefore \delta(\alpha) = \alpha - \beta + \pi - 2\beta + \alpha - \beta = \underline{\pi + 2\alpha - 4\beta} \quad \text{vsv.}$$

b) Snell $\sin\alpha = n \cdot \sin\beta \Rightarrow \beta = \arcsin\left(\frac{\sin\alpha}{n}\right)$

$$\therefore \delta(\alpha) = \pi + 2\alpha - 4 \cdot \arcsin\left(\frac{\sin\alpha}{n}\right)$$

c) $$\frac{\partial\delta}{\partial\alpha} = 2 - 4 \cdot \underbrace{\frac{\partial}{\partial\alpha}\left(\arcsin\left(\frac{\sin\alpha}{n}\right)\right)}$$

$$\frac{\partial}{\partial\alpha}\left(\arcsin\left(\frac{\sin\alpha}{n}\right)\right) = \frac{\frac{1}{n}\cdot\cos\alpha}{\sqrt{1-\left(\frac{\sin\alpha}{n}\right)^2}}$$

$$2 - \frac{4}{n}\frac{\cos\alpha}{\sqrt{1-\left(\frac{\sin\alpha}{n}\right)^2}} = 0 \Rightarrow \cancel{4}\left(1-\left(\frac{\sin\alpha}{n}\right)^2\right) = \frac{16}{n^2}\cdot\cos^2\alpha$$

$$\Rightarrow n^2 - \sin^2\alpha = 4\cos^2\alpha \Rightarrow \cos^2\alpha = \frac{1}{3}(n^2-1)$$

$$\underline{\underline{\alpha = \arccos\left(\sqrt{\frac{n^2-1}{3}}\right)}} = 59{,}3911\ldots \qquad \alpha = 59{,}4° \text{ för vatten}$$

$\uparrow$ $n = 4/3$ vsv.

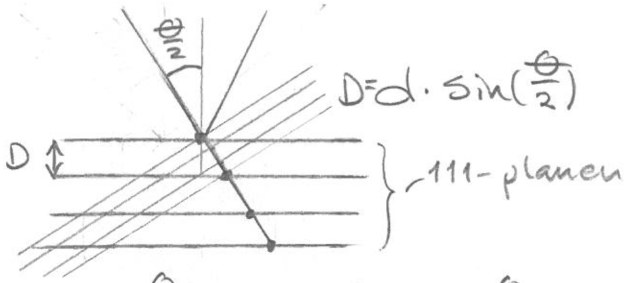
d) $\alpha = \arccos\left(\sqrt{\frac{n^2-1}{3}}\right)$, Första ordningen möjlig om

$n^2 > 1$ och $n^2 - 1 \leq 3 \Rightarrow 1 < n \leq 2$

Dvs. $n = \frac{12}{5}$ kan inte ge en första ordn. båge.

#18

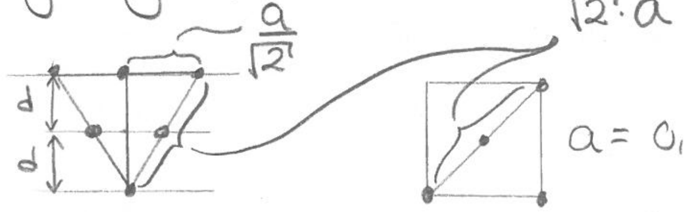
6.



$$\Delta r = 2D\cdot\cos\left(\frac{\theta}{2}\right) = 2\cdot d\cdot\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right) = d\cdot\sin\theta$$

$\Delta r = m\cdot\lambda \qquad (m=1)$

Enligt graf $\theta = 50°$ $(m=1)$



$a = 0{,}3524$ nm

Enligt kristallfigur: $(2d)^2 + \left(\frac{a}{\sqrt{2}}\right)^2 = (\sqrt{2}\cdot a)^2 \Rightarrow$

$$\Rightarrow (2d)^2 = a^2\left(2-\frac{1}{2}\right) \Rightarrow d = \frac{\overset{0{,}3524\,nm}{a}\cdot\sqrt{3/2}}{2} = 0{,}2158 \text{ nm}$$

$$\Delta r = d\cdot\sin\theta \Rightarrow \lambda = \frac{d\cdot\sin\theta}{m} = 0{,}16531\ldots$$

$\therefore \lambda = 0{,}165$ nm

b) Enligt de Broglie $\lambda = \frac{h}{p} = \frac{h}{m\cdot v}$ där $v = 4{,}35\cdot10^6$ m/s i experimentet

Med $m = 9{,}11\cdot10^{-31}$ kg

$\lambda = \underline{\underline{0{,}167 \text{ nm}}}$ dvs. ungefär vad Davissen, Germer fick.