a) $\bar{E}(x,t) = E_{0y}\sin(kx-\omega t)\hat{y} + E_{0z}\sin(kx-\omega t+\phi_0)\hat{z}$

i Linjärpolariserat $\underline{\phi_0 = m\pi \quad (m \in \mathbb{Z})}$

$$\Rightarrow \bar{E}(x,t) = \left(E_{0y}\hat{y} \pm E_{0z}\hat{z}\right)\sin(kx-\omega t)$$

ii Cirkulärpolariserat $\underline{\phi_0 = (m+\tfrac{1}{2})\pi \quad (m \in \mathbb{Z})}$

och $\underline{E_{0y} = E_{0z} \equiv E_0}$

$$\Rightarrow \bar{E}(x,t) = E_0\left(\sin(kx-\omega t)\hat{y} \pm \cos(kx-\omega t)\hat{z}\right)$$

b)

i $E_t = E_0 \cdot \cos\theta \quad \therefore \cos\theta = 0{,}8 \Rightarrow \theta = \arccos(0{,}8) = 36{,}869\ldots^\circ$

Svar: $\theta = 36{,}87^\circ$

ii $I_t = I_0\cos^2\theta$ ("Malus lag")

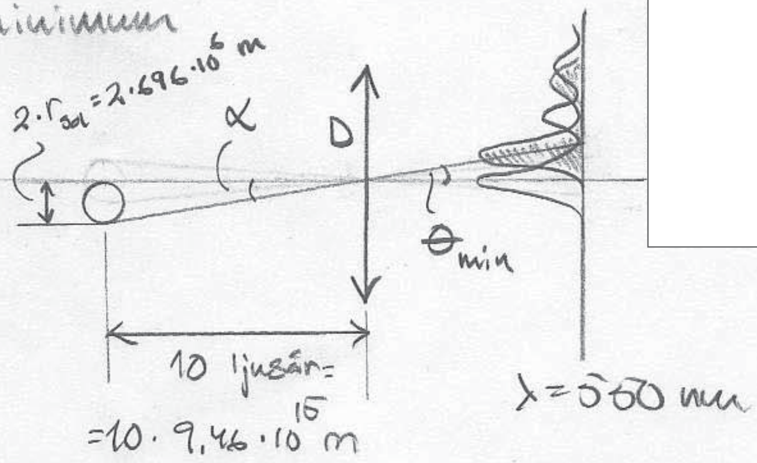
$\therefore \cos^2\theta = 0{,}8 \Rightarrow \theta = \arccos(\sqrt{0{,}8}) = 26{,}565\ldots^\circ$

$\theta = \arccos(-\sqrt{0{,}8}) = 153{,}434\ldots^\circ$

Svar: $\theta = 26{,}56^\circ$ eller $153{,}43^\circ$

a, i, upplösas (alt: särskiljas)
ii, första iii, minimum

b, $\alpha > \theta_{min} = \dfrac{1{,}22\,\lambda}{D}$



på gränsen $\alpha = \theta_{min}$

$$\alpha \approx \tan\alpha = \frac{2 \cdot r_{sol}}{10 \text{ ljusår}} = 1{,}47 \cdot 10^{-8}$$

$$D = \frac{1{,}22 \cdot \lambda}{\alpha} = \frac{1{,}22 \cdot 550 \cdot 10^{-9}}{1{,}47 \cdot 10^{-8}} = 45{,}64\ldots \text{ m}$$

Svar: 45,6 m

c, $\lambda = 10^{-2}$ m
$D = 305$ m

$$\theta_{min} = \frac{1{,}22 \cdot \lambda}{D} = \frac{1{,}22 \cdot 10^{-2}}{305} = 4 \cdot 10^{-5}$$

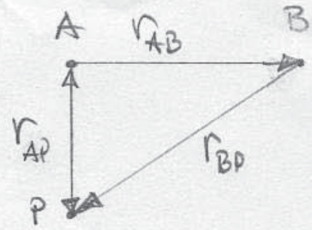
Nej: $\theta_{min} \gg \alpha$

---

c, nödv. diameter D

$$D = \frac{1{,}22 \cdot 10^{-2}}{1{,}47 \cdot 10^{-8}} = 829932 \text{ m!}$$

4.



$r_{AP} = 2.0$ m $I_0 = 10^{-12}$ W/m²

$L_A = 90$ dB $L_B = 84{,}7$ dB

$f_A = f_B = 1$ kHz

b) $L = 10 \cdot \lg \frac{I_P^A}{I_0} \Rightarrow 10^{\left(\frac{L}{10}\right)} = \frac{I_P^A}{I_0}$

$\therefore I_P^A = I_0 \cdot 10^9 = \underline{\underline{10^{-3}\ \text{W/m}^2}}$

PH 5.3

$I = \frac{\langle \frac{dW}{dt} \rangle}{A} = \frac{P}{A} \Rightarrow P = I_P^A \cdot \underbrace{A}_{4\pi r_{AP}^2}$

c) $\therefore P = \underbrace{10^{-3}}_{I_P} \cdot 4\pi \cdot 4 = 16\pi \cdot 10^{-3} = 0{,}05026\ldots$ W

$\underline{\underline{P = 50\ \text{mW}}}$

d) $I_P^B = I_0 \cdot 10^{8.47}$

$A = \frac{P}{I_P^B} \Rightarrow r_{BP} = \sqrt{\frac{\overbrace{P}^{I_P^A \cdot 4\pi r_{AP}^2}}{4\pi I_P^B}} = 3{,}681\ldots$ m

$r_{AB} = \sqrt{r_{BP}^2 - r_{AP}^2} = 3{,}0909167\ldots$ m

$\therefore \underline{\underline{r_{AB} = 3{,}1\ \text{m}}}$

$\left(\frac{I_P^A}{I_P^B} = \frac{r_{BP}^2}{r_{AP}^2} \quad r_{BP} = r_{AP}\sqrt{\frac{I_P^A}{I_P^B}} = r_{AP}\sqrt{10^{0{,}53}} = 3{,}681\ldots \text{ m} \right)$

a) $\beta_4 = 10 \cdot \lg\left(\frac{4I}{I_0}\right)$ $\beta_1 = 10 \lg\left(\frac{I}{I_0}\right)$

$\beta_4 - \beta_1 = 10 \cdot \lg\left(\frac{4I}{I}\right) = 6{,}0205\ldots$ dB Svar: Minskar med $\underline{\underline{6\text{dB}}}$

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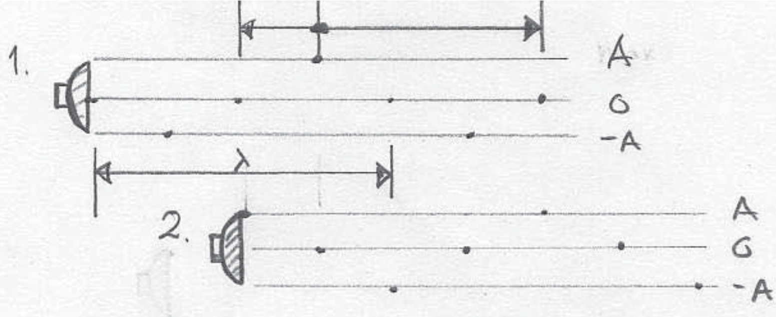
2012-01-09
4a)



$\lambda = 2{,}0\ \text{m}$

Flytta 1. $\frac{\lambda}{4} = 0{,}5$ m åt vänster (alt. iv)

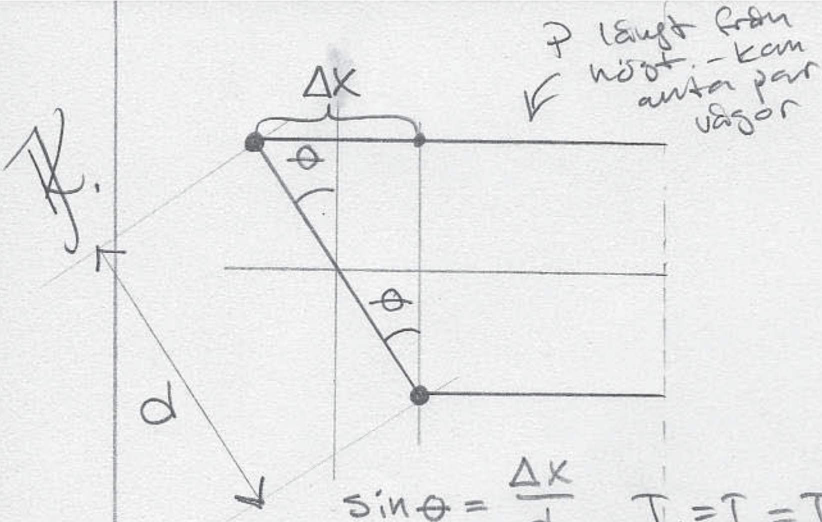
eller

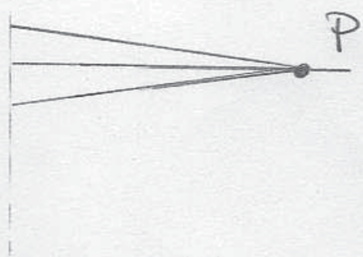
Flytta 1. $\frac{3}{4} \cdot \lambda = 1{,}5$ m åt höger (alt. finns ej)

eller

...

Vågfysik
2012-01-09
4bc)





$$\sin\theta = \frac{\Delta x}{d}, \quad I_1 = I_2 = I, \quad f_1 = f_2 = f = \frac{v}{\lambda} \quad (\lambda_1 = \lambda_2 = \lambda)$$

a)

$$I_{tot} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cdot\cos(\Delta\delta) = 2I + 2I\cdot\cos(\Delta\delta)$$

$$\Delta\delta = \delta_2 - \delta_1 = -qx_2 + \phi_2 - (-qx_1 + \phi_1) = \underset{\frac{2\pi}{\lambda}}{q}\underbrace{(x_1 - x_2)}_{\Delta x} + \overbrace{(\phi_2 - \phi_1)}^{=0 \text{ Synkrona}}$$

$$\therefore \ \Delta\delta = \frac{2\pi}{\lambda}\cdot d\cdot\sin\theta$$

$$I_{tot} = 2I\left(1 + \cos\left[\frac{2\pi}{\lambda}\cdot d\cdot\sin\theta\right]\right)$$

b) • Max $\underline{\underline{I_{tot} = 4I}}$ då $\cos\left[\frac{2\pi}{\lambda}\cdot d\cdot\sin\theta\right] = 1$

$\lambda = d \quad 2\pi\cdot\sin\theta = 2\pi\cdot m \quad (m\in\mathbb{Z})$ ← ej alla m möjliga

$\Rightarrow \quad \sin\theta = m \quad (m = -1, 0, +1)$

$\sin\theta = -1$ då $\underline{\underline{\theta = 270^\circ}}$

$\sin\theta = 0$ då $\underline{\underline{\theta = 0^\circ, 180^\circ, 360^\circ}}$

$\sin\theta = +1$ då $\underline{\underline{\theta = 90^\circ}}$

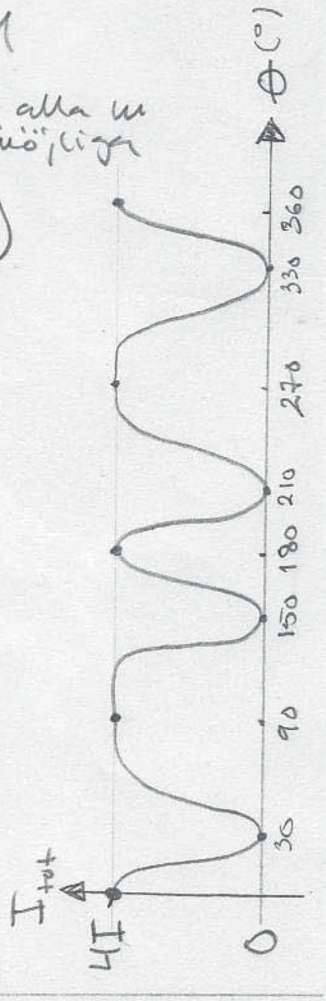
• Min $\underline{\underline{I_{tot} = 0}}$ då $\left[\cos\frac{2\pi}{\lambda}\cdot d\sin\theta\right] = -1$

$\lambda = d \quad 2\pi\sin\theta = \pi(2m+1) \quad (m\in\mathbb{Z})$

$\Rightarrow \quad \sin\theta = m + \frac{1}{2} \quad (m = -1, 0)$

$\sin\theta = -\frac{1}{2}$ då $\underline{\underline{\theta = 210^\circ, 330^\circ}}$

$\sin\theta = +\frac{1}{2}$ då $\underline{\underline{\theta = 30^\circ, 150^\circ}}$



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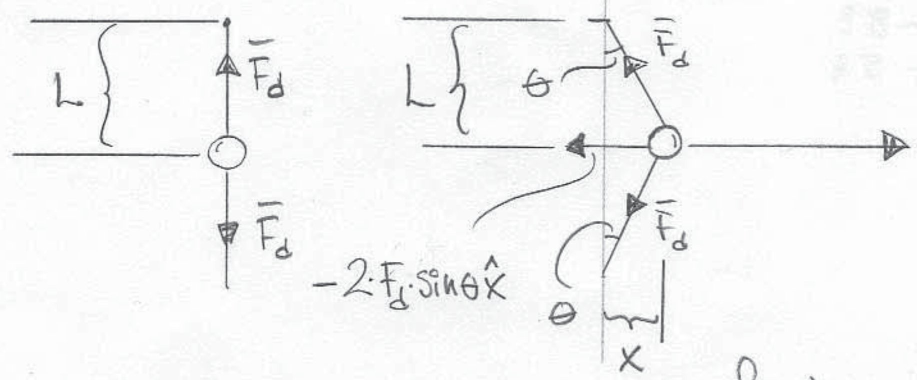
2012-01-09
5.

a)



$$\left.\begin{aligned}\bar{F} &= m\cdot\bar{a} = m\cdot\ddot{x}\,\hat{x}\\ \bar{F} &= -2F_d\cdot\sin\theta\,\hat{x}\end{aligned}\right\}\quad \underbrace{-2F_d\cdot\sin\theta}_{f(x)} = m\ddot{x}$$

ej harmonisk

b) Små θ: $\sin\theta \simeq \tan\theta = \dfrac{x}{L}$

$$\Rightarrow\; -\frac{2F_d}{L}\cdot x = m\ddot{x} \;\Rightarrow\; \ddot{x} + \frac{2F_d}{m\cdot L}\cdot x = 0$$

harmonisk

c) $r^2 + \dfrac{2F_d}{m\cdot L} = 0 \;\Rightarrow\; r \pm i\sqrt{\dfrac{2F_d}{m\cdot L}}$

$$x(t) = A\cos\sqrt{\frac{2F_d}{mL}}\cdot t + B\sin\sqrt{\frac{2F_d}{mL}}\cdot t$$

$$\dot{x}(t) = -A\sqrt{\frac{2F_d}{mL}}\cdot\sin\sqrt{\frac{2F_d}{mL}} + B\sqrt{\frac{2F_d}{mL}}\cdot\cos\sqrt{\frac{2F_d}{mL}}\cdot t$$

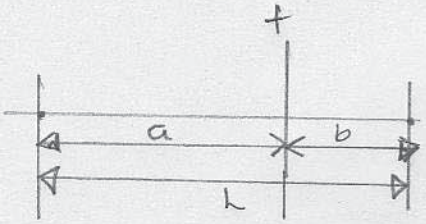
$$x(0) = x_0 \Rightarrow A = x_0,\quad \dot{x}(0) = 0 \Rightarrow B = 0$$

$$x(t) = x_0\cdot\cos\Big(\underbrace{\sqrt{\frac{2F_d}{mL}}}_{\omega}\cdot t\Big)$$

$$\omega = \sqrt{\frac{2F_d}{m\cdot L}} = \sqrt{\frac{2\cdot 250}{1{,}5\cdot 1{,}8}} \approx 13{,}6\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = 2{,}16\ \text{Hz}$$

$$T = \frac{1}{f} = 0{,}46\ \text{s}$$



Sträckorna a och b har förhållandet som ger gyllene snittet

K.

a, givet att $\frac{a+b}{a} = \frac{a}{b} = \varphi$, $a+b = L$

uttryck för f via Gauss linsekv. $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$

$$\frac{L}{a} = \varphi \Rightarrow a = \frac{L}{\varphi} \quad , \quad \frac{a}{b} = \varphi \Rightarrow b = \frac{a}{\varphi} = \frac{L}{\varphi^2}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \Rightarrow \frac{\varphi}{L} + \frac{\varphi^2}{L} = \frac{1}{f} \Rightarrow \underline{\underline{f = \frac{L}{(\varphi+1)\varphi}}} = \underline{\underline{\frac{L}{\varphi^3}}}$$

Förhållandet mellan a och b ger också att

$$a = \varphi b \Rightarrow \frac{a+b}{a} = \frac{(\varphi+1)\cancel{b}}{\varphi \cancel{b}} = \varphi \Rightarrow (\varphi+1) = \varphi^2$$

$$\text{alt. } b = \frac{a}{\varphi} \Rightarrow \frac{a+b}{a} = \frac{a(1+\frac{1}{\varphi})}{a} = \varphi \Rightarrow (\varphi - 1) = \frac{1}{\varphi}$$

$\frac{1}{\varphi} \equiv \phi$

Förstoringen $\underline{\underline{M}} = -\frac{b}{a} = \underline{\underline{-\frac{1}{\varphi} = (1-\varphi)}}$

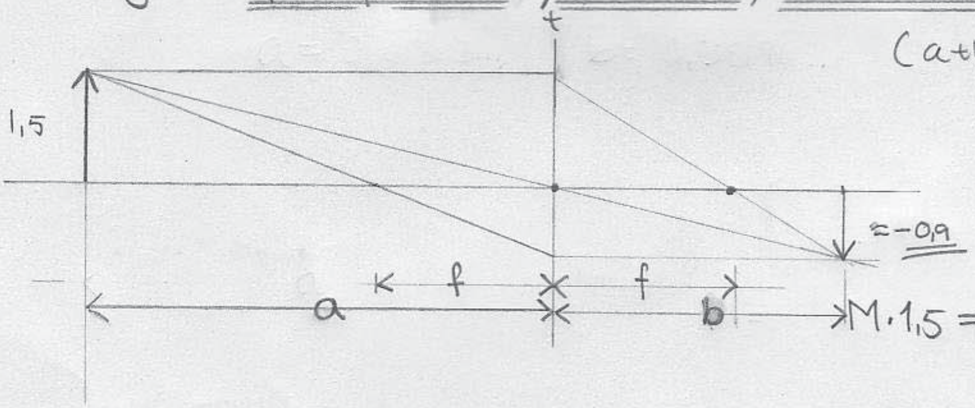
b, $L = 1$ m givet, φ?

Från t.ex. $\varphi + 1 = \varphi^2 \Rightarrow \varphi^2 - \varphi - 1 = 0 \Rightarrow \varphi = \frac{1 (\pm) \sqrt{1^2+4}}{2} = \frac{1+\sqrt{5}}{2} = 1{,}61803\ldots$

Vilket ger $\underline{\underline{f = 0{,}23606\ldots \text{ m}}}$, $\underline{\underline{M = -0{,}6183\ldots}}$, $\underline{\underline{a = 0{,}61803\ldots \text{ m}}}$, $\underline{\underline{b = 0{,}38196\ldots \text{ m}}}$

(a+b=1 ok!)

c,



$M \cdot 1{,}5 = \underline{\underline{-0{,}9270\ldots}}$ OK!

TFYA59 (MED)

Vågfysik
2012-01-09
6.I (se 6. ovan)

6.II

$$\frac{d}{dx}(I(x)) = -\alpha I(x) \qquad I(0) = I_0$$

a, $I(x) = I_0 e^{-\alpha \cdot x}$, $I(x_{1/2}) = \frac{I_0}{2}$ $\Big\}$ $\frac{I_0}{2} = I_0 e^{-\alpha \cdot x_{1/2}}$

$$\Rightarrow \ln\frac{1}{2} = -\alpha \cdot x_{\frac{1}{2}} \Rightarrow \alpha = \frac{\ln 2}{x_{1/2}}$$

$$\alpha_{ben} = \frac{\ln 2}{1{,}9} = 0{,}3648\ldots \text{ cm}^{-1}$$

$$\alpha_{muskel} = \frac{\ln 2}{3{,}9} = 0{,}1777\ldots \text{ cm}^{-1}$$

Svar: $\alpha_{ben} = 0{,}36 \text{ cm}^{-1}$, $\alpha_{muskel} = 0{,}18 \text{ cm}^{-1}$

b, $I(d) = I_0 e^{-1} \Rightarrow -\alpha \cdot d = -1 \Rightarrow d = \frac{1}{\alpha}$

$$d_{ben} = \frac{1}{\alpha_{ben}} = 2{,}741\ldots \text{ cm}$$

$$d_{muskel} = \frac{1}{\alpha_{muskel}} = 5{,}626\ldots \text{ cm}$$

Svar: $d_{ben} = 2{,}74 \text{ cm}$, $d_{muskel} = 5{,}63 \text{ cm}$