22.1. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b). It is symmetrical with the m = 2 fringes on both sides of and equally distant from the central maximum.
Solve: The bright fringes occur at angles θ_m such that

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \sin\theta_2 = \frac{2\left(500\times10^{-9}\ \text{m}\right)}{\left(50\times10^{-6}\ \text{m}\right)} = 0.02 \Rightarrow \theta_2 = 0.020\ \text{rad} = 0.020\ \text{rad}\times\frac{180°}{\pi\ \text{rad}} = 1.15°$$

22.2. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b). It is symmetrical, with the m = 2 fringes on both sides of and equally distant from the central maximum.
Solve: The two paths from the two slits to the m = 2 bright fringe differ by $\Delta r = r_2 - r_1$, where

$$\Delta r = m\lambda = 2\lambda = 2(500\text{ nm}) = 1000\text{ nm}$$

Thus, the position of the m = 2 bright fringe is 1000 nm farther away from the more distant slit than from the nearer slit.

22.3. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b).
Solve: The bright fringes are located at positions given by Equation 22.4, $d\sin\theta_m = m\lambda$. For the $m = 3$ bright orange fringe, the interference condition is $d\sin\theta_3 = 3(600\times10^{-9}\text{ m})$. For the $m = 4$ bright fringe the condition is $d\sin\theta_4 = 4\lambda$. Because the position of the fringes is the same,

$$d\sin\theta_3 = d\sin\theta_4 = 4\lambda = 3(600\times10^{-9}\text{ m}) \Rightarrow \lambda = \tfrac{3}{4}(600\times10^{-9}\text{ m}) = 450\text{ nm}$$

22.4. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b).
Solve: The formula for fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow 1.8\times10^{-3}\ \text{m} = \left(600\times10^{-9}\ \text{m}\right)\frac{L}{d} \Rightarrow \frac{L}{d} = 3000$$

The wavelength is now changed to 400 nm, and L/d, being a part of the experimental setup, stays the same. Applying the above equation once again,

$$\Delta y = \frac{\lambda L}{d} = \left(400\times10^{-9}\ \text{m}\right)\left(3000\right) = 1.2\ \text{mm}$$

22.5. Visualize: The fringe spacing for a double slit pattern is $\Delta y = \dfrac{\lambda L}{d}$. We are given $L = 2.0\ \text{m}$ and $\lambda = 600\ \text{nm}$. We also see from the figure that $\Delta y = \tfrac{1}{3}\text{cm}$.

Solve: Solve the equation for *d*.

$$d = \frac{\lambda L}{\Delta y} = \frac{(600\times10^{-9}\ \text{m})(2.0\ \text{m})}{\tfrac{1}{3}\times10^{-2}\ \text{m}} = 0.36\ \text{mm}$$

Assess: 0.36 mm is a typical slit spacing.

22.6. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b).
Solve: The fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(589\times10^{-9}\text{ m})(150\times10^{-2}\text{ m})}{4.0\times10^{-3}\text{ m}} = 0.22\text{ mm}$$

22.7. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b).
Solve: The dark fringes are located at positions given by Equation 22.9:

$$y'_m = \left(m + \tfrac{1}{2}\right)\frac{\lambda L}{d} \qquad m = 0, 1, 2, 3, \ldots$$

$$\Rightarrow\ y'_5 - y'_1 = \left(5 + \tfrac{1}{2}\right)\frac{\lambda L}{d} - \left(1 + \tfrac{1}{2}\right)\frac{\lambda L}{d} \Rightarrow 6.0 \times 10^{-3}\ \text{m} = \frac{4\lambda(60 \times 10^{-2}\ \text{m})}{0.20 \times 10^{-3}\ \text{m}} \Rightarrow \lambda = 500\ \text{nm}$$

22.8. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 22.3(b).
Solve: In a span of 12 fringes, there are 11 gaps between them. The formula for the fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow \left(\frac{52\times10^{-3}\text{ m}}{11}\right) = \frac{(633\times10^{-9}\text{ m})(3.0\text{ m})}{d} \Rightarrow d = 0.40\text{ mm}$$

Assess: This is a reasonable distance between the slits, ensuring $d/L = 1.34\times10^{-4} << 1$.

22.9. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 22.8.
Solve: The bright constructive-interference fringes are given by Equation 22.15:

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots$$

$$\Rightarrow \sin\theta_1 = \frac{(1)(550\times10^{-9}\text{ m})}{(1.0\times10^{-2}\text{ m})/1000} = 0.055 \Rightarrow \theta_1 = 3.2°$$

Likewise, $\sin\theta_2 = 0.110$ and $\theta_2 = 6.3°$.

22.10. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m determined by Equation 22.15.

Solve: We have

$$d\sin\theta_m = m\lambda \qquad m = 0,\ 1,\ 2,\ 3,\ \ldots \Rightarrow\ d\sin 20.0° = 1\lambda \ \text{ and } \ d\sin\theta_2 = 2\lambda$$

Dividing these two equations,

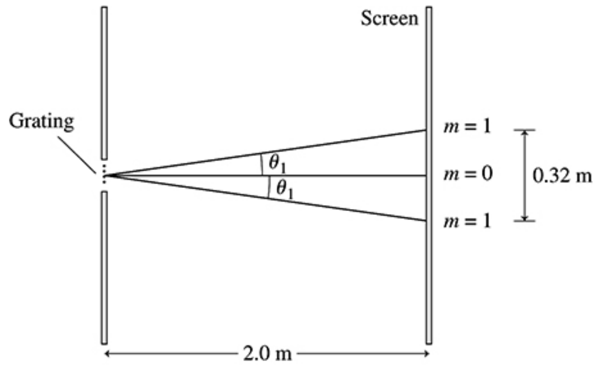
$$\sin\theta_2 = 2\sin 20.0° = 0.6840 \ \Rightarrow\ \theta_2 = 43.2°$$

22.11. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 22.8.
Solve: The bright constructive-interference fringes are given by Equation 22.15:

$$d\sin\theta_m = m\lambda \Rightarrow d = \frac{m\lambda}{\sin\theta_m} = \frac{(2)(600\times10^{-9}\text{ m})}{\sin(39.5°)} = 1.89\times10^{-6}\text{ m}$$

The number of lines in per millimeter is $(1\times10^{-3}\text{ m})/(1.89\times10^{-6}\text{ m}) = 530$.

22.12. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 22.8.



Solve: The bright fringes are given by Equation 22.15:

$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots \Rightarrow d \sin\theta_1 = (1)\lambda \Rightarrow d = \lambda/\sin\theta_1$$

The angle θ_1 can be obtained from geometry as follows:

$$\tan\theta_1 = \frac{(0.32 \text{ m})/2}{2.0 \text{ m}} = 0.080 \Rightarrow \theta_1 = \tan^{-1} (0.080)\ 4.57°$$

Using $\sin\theta_1 = \sin 4.57° = 0.07968$,

$$d = \frac{633 \times 10^{-9} \text{ m}}{0.07968} = 7.9\ \mu\text{m}$$

22.13. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram of Figure 22.8.
Solve: The bright interference fringes are given by

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

The slit spacing is $d = 1\ \text{mm}/500 = 2.00\times10^{-6}\ \text{m}$ and $m = 1$. For the red and blue light,

$$\theta_{1\ \text{red}} = \sin^{-1}\left(\frac{656\times10^{-9}\ \text{m}}{2.00\times10^{-6}\ \text{m}}\right) = 19.15° \qquad \theta_{1\ \text{blue}} = \sin^{-1}\left(\frac{486\times10^{-9}\ \text{m}}{2.00\times10^{-6}\ \text{m}}\right) = 14.06°$$

The distance between the fringes, then, is $\Delta y = y_{1\ \text{red}} - y_{1\ \text{blue}}$ where

$$y_{1\ \text{red}} = (1.5\ \text{m})\tan 19.15° = 0.521\ \text{m}$$
$$y_{1\ \text{blue}} = (1.5\ \text{m})\tan 14.06° = 0.376\ \text{m}$$

So, $\Delta y = 0.145\ \text{m} = 14.5\ \text{cm}$.

22.14. Model: Assume the screen is centered behind the slit. We actually want to solve for *m*, but given the other data, it is unlikely that we will get an integer from the equations for the edge of the screen, so we will have to truncate our answer to get the largest order fringe on the screen.
Visualize: Refer to Figure 22.7. Use Equation 22.15: $d\sin\theta_m = m\lambda$, and Equation 22.16: $y_m = L\tan\theta_m$. We are given $\lambda = 510$ nm, $L = 2.0$ m, and $d = \frac{1}{500}$mm. As mentioned above, we are not guaranteed that a bright fringe will occur exactly at the edge of the screen, but we will kind of assume that one does and set $y_m = 1.0$ m; if we do not get an integer for *m* then the fringe was not quite at the edge of the screen and we will truncate our answer to get an integer *m*.
Solve: Solve Equation 22.16 for θ_m and insert it in Equation 22.15.

$$\theta_m = \tan^{-1}\frac{y_m}{L}$$

Solve Equation 22.15 for m.

$$m = \frac{d}{\lambda}\sin\theta_m = \frac{d}{\lambda}\sin\left(\tan^{-1}\frac{y_m}{L}\right) = \frac{\frac{1}{500}\text{ mm}}{510\text{ nm}}\sin\left(\tan^{-1}\frac{1.0\text{ m}}{2.0\text{ m}}\right) = 1.8$$

Indeed, we did not get an integer, so truncate 1.8 to get $m = 1$. This means we will see three fringes, one for $m = 0$, and one on each side for $m = \pm 1$.
Assess: Our answer fits with the statement in the text: "Practical gratings, with very small values for d, display only a few orders."

22.15. Model: A narrow single slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 22.14.
Solve: The minima occur at positions

$$y_p = p\frac{L\lambda}{a}$$

So $\Delta y = y_2 - y_1 = \frac{2\lambda L}{a} - \frac{1\lambda L}{a} = \frac{\lambda L}{a} \Rightarrow a = \frac{\lambda L}{\Delta y} = \frac{(633\times10^{-9}\text{ m})(1.5\text{ m})}{0.00475\text{ m}} = 2.0\times10^{-4}\text{ m} = 0.20\text{ mm}$

22.16. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 22.14.
Solve: The width of the central maximum for a slit of width $a = 200\lambda$ is

$$w = \frac{2\lambda L}{a} = \frac{2\lambda(2.0\text{ m})}{200\lambda} = 20\text{ mm}$$

22.17. Model: A narrow slit produces a single-slit diffraction pattern.

Visualize: The intensity pattern for single-slit diffraction will look like Figure 22.14.

Solve: Angle $\theta = 0.70° = 0.0122$ rad is a small angle (<< 1 rad). Thus we use Equation 22.20 to find the wavelength of light. The angles of the minima of intensity are

$$\theta_p = p\frac{\lambda}{a} \qquad p = 1, 2, 3, \kappa$$

$$\Rightarrow \lambda = \frac{a\theta_p}{p} = \frac{(0.10\times10^{-3}\ \text{m})(0.0122\ \text{rad})}{2} = 610\ \text{nm}$$

22.18. Visualize: Use Equation 22.22: $w = \frac{2\lambda L}{a}$. We are given $\lambda = 600$ nm and $L = 2.0$ m. We see from the figure that $w = 1.0$ cm.
Solve: Solve the equation for a.

$$a = \frac{2\lambda L}{w} = \frac{2(600 \times 10^{-9}\ \text{m})(2.0\ \text{m})}{0.010\ \text{m}} = 0.24\ \text{mm}$$

Assess: 0.24 mm is in the range of typical slit widths.

22.19. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 22.14.
Solve: The width of the central maximum for a slit of width $a = 200\lambda$ is

$$w = \frac{2\lambda L}{a} = \frac{2\left(500\times10^{-9}\ \text{m}\right)\left(2.0\ \text{m}\right)}{0.0005\ \text{m}} = 0.0040\ \text{m} \ = \ 4.0\ \text{mm}$$

22.20. **Model:** The spacing between the two buildings is like a single slit and will cause the radio waves to be diffracted.

Solve: Radio waves are electromagnetic waves that travel with the speed of light. The wavelength of the 800 MHz waves is

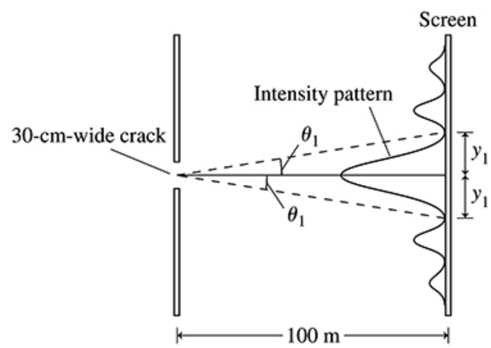
$$\lambda = \frac{3\times 10^8 \text{ m/s}}{800\times 10^6 \text{ Hz}} = 0.375 \text{ m}$$

To investigate the diffraction of these waves through the spacing between the two buildings, we can use the general condition for complete destructive interference: $a\sin\theta_p = p\lambda$ (p = 1, 2, 3, …) where a is the spacing between the buildings. Because the width of the central maximum is defined as the distance between the two p = 1 minima on either side of the central maximum, we will use p = 1 and obtain the angular width $\Delta\theta = 2\theta_1$ from

$$a\sin\theta_1 = \lambda \Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{0.375 \text{ m}}{15 \text{ m}}\right) = 1.43^\circ$$

Thus, the angular width of the wave after it emerges from between the buildings is $\Delta\theta = 2(1.43^\circ) = 2.86^\circ \approx 2.9^\circ$.

22.21. Model: The crack in the cave is like a single slit that causes the ultrasonic sound beam to diffract.
Visualize:



Solve: The wavelength of the ultrasound wave is

$$\lambda = \frac{340 \text{ m/s}}{30 \text{ kHz}} = 0.0113 \text{ m}$$

Using the condition for complete destructive interference with $p = 1$,

$$a\sin\theta_1 = \lambda \Rightarrow \theta_1 = \sin^{-1}\left(\frac{0.0113 \text{ m}}{0.30 \text{ m}}\right) = 2.165°$$

From the geometry of the diagram, the width of the sound beam is

$$w = 2y_1 = 2(100 \text{ m} \times \tan\theta_1) = 200 \text{ m} \times \tan 2.165° = 7.6 \text{ m}$$

Assess: The small-angle approximation is almost always valid for the diffraction of light, but may not be valid for the diffraction of sound waves, which have a much larger wavelength.

22.22. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.

Solve: The width of the central maximum for a circular aperture of diameter D is

$$w = \frac{2.44\lambda L}{D} = \frac{(2.44)(500\times10^{-9}\text{ m})(2.0\text{ m})}{0.50\times10^{-3}\text{ m}} = 4.9\text{ mm}$$

22.23. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Visualize: The intensity pattern will look like Figure 22.15.
Solve: According to Equation 22.23, the angle that locates the first minimum in intensity is

$$\theta_1 = \frac{1.22\lambda}{D} = \frac{1.22(2.5\times10^{-6}\ \text{m})}{0.20\times10^{-3}\ \text{m}} = 0.01525\ \text{rad} = 0.874°$$

These should be rounded to 0.015 rad = 0.87°.

22.24. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.

Visualize: The intensity pattern will look like Figure 22.15.

Solve: From Equation 22.24, the diameter of the circular aperture is

$$D = \frac{2.44\lambda L}{w} = \frac{2.44(633\times10^{-9}\text{ m})(4.0\text{ m})}{2.5\times10^{-2}\text{ m}} = 0.25\text{ mm}$$

22.25. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Visualize: The intensity pattern will look like Figure 22.15.
Solve: From Equation 22.24,

$$L = \frac{Dw}{2.44\lambda} = \frac{(0.12\times10^{-3}\ \text{m})(1.0\times10^{-2}\ \text{m})}{2.44(633\times10^{-9}\ \text{m})} = 78\ \text{cm}$$

22.26. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 22.20.
Solve: From Equation 22.33, the wavelength is

$$\lambda = \frac{2\Delta L_2}{\Delta m} = \frac{2(100\times10^{-6}\ \text{m})}{500} = 4.0\times10^{-7}\ \text{m} = 400\ \text{nm}$$

22.27. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 22.20.
Solve: From Equation 22.33, the number of fringe shifts is

$$\Delta m = \frac{2\Delta L_2}{\lambda} = \frac{2(1.00\times10^{-2}\ \text{m})}{656.45\times10^{-9}\ \text{m}} = 30,467$$

22.28. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 22.20.
Solve: From Equation 22.33, the distance the mirror moves is

$$\Delta L_2 = \frac{\Delta m\lambda}{2} = \frac{(33{,}198)(602.446\times10^{-9}\ \text{m})}{2} = 0.0100000\ \text{m} = 1.00000\ \text{cm}$$

Assess: Because the wavelength is known to 6 significant figures and the fringes are counted exactly, we can determine ΔL to 6 significant figures.

22.29. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 22.20.
Solve: For sodium light of the longer wavelength (λ_1) and of the shorter wavelength (λ_2),

$$\Delta L = m\frac{\lambda_1}{2} \qquad \Delta L = (m+1)\frac{\lambda_2}{2}$$

We want the same path difference $2(L_2 - L_1)$ to correspond to one extra wavelength for the sodium light of shorter wavelength (λ_2) . Thus, we combine the two equations to obtain:

$$m\frac{\lambda_1}{2} = (m+1)\frac{\lambda_2}{2} \Rightarrow m(\lambda_1 - \lambda_2) = \lambda_2 \Rightarrow m = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{589.0 \text{ nm}}{589.6 \text{ nm} - 589.0 \text{ nm}} = 981.67 \cong 982$$

Thus, the distance by which M_2 is to be moved is

$$\Delta L = m\frac{\lambda_1}{2} = 982\left(\frac{589.6 \text{ nm}}{2}\right) = 0.2895 \text{ mm}$$

22.30. **Model:** Two closely spaced slits produce a double-slit interference pattern with the intensity graph looking like Figure 22.3(b). The intensity pattern due to a single slit diffraction looks like Figure 22.14. Both the spectra consist of a central maximum flanked by a series of secondary maxima and dark fringes.

Solve: **(a)** The light intensity shown in Figure P22.30 corresponds to a double-slit aperture. This is because the fringes are equally spaced and the decrease in intensity with increasing fringe order occurs slowly.

(b) From Figure P22.30, the fringe spacing is $\Delta y = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$. Therefore,

$$\Delta y = \frac{\lambda L}{d}$$

$$\Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(6.00 \times 10^{-9} \text{ m})(2.5 \text{ m})}{0.010 \text{ m}} = 0.15 \text{ mm}$$

22.31. Model: Two closely spaced slits produce a double-slit interference pattern with the intensity graph looking like Figure 22.3(b). The intensity pattern due to a single slit diffraction looks like Figure 22.14. Both the spectra consist of a central maximum flanked by a series of secondary maxima and dark fringes.

Solve: **(a)** The light intensity shown in Figure P22.31 corresponds to a single slit aperture. This is because the central maximum is twice the width and much brighter than the secondary maximum.

(b) From Figure P22.31, the separation between the central maximum and the first minimum is $y_1 = 1.0\text{ cm} = 1.0 \times 10^{-2}\text{ m}$. Therefore, using the small-angle approximation, Equation 22.21 gives the condition for the dark minimum:

$$y_p = \frac{pL\lambda}{d} \Rightarrow a = \frac{L\lambda}{y_1} = \frac{(2.5\text{ m})(600 \times 10^{-9}\text{ m})}{1.0 \times 10^{-2}\text{ m}} = 0.15\text{ mm}$$

22.32. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference fringes are equally spaced on both sides of the central maximum. The interference pattern looks like Figure 22.3(b).
Solve: In the small-angle approximation

$$\Delta\theta = \theta_{m+1} - \theta_m = (m+1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}$$

Since $d = 200\lambda$, we have

$$\Delta\theta = \frac{\lambda}{d} = \frac{1}{200}\text{ rad} = 0.286°$$

22.33. Model: Two closely spaced slits produce a double-slit interference pattern.

Solve: The light intensity of a double-slit interference pattern at a position y on the screen is

$$I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right) = 4I_1 \cos^2\left(\frac{y}{\Delta y}\pi\right)$$

where $\Delta y = \lambda L / d = 4.0 \text{ mm}$ is the fringe spacing.

Using this value for $\lambda L/d$, we can find the position on the interference pattern where $I_{\text{double}} = I_1$ as follows:

$$4I_1 \cos^2\left(\frac{\pi}{4.0\times10^{-3}\text{ m}} y\right) = I_1 \Rightarrow \left(\frac{\pi}{4.0\times10^{-3}\text{ m}}\right) y = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}\text{ rad} \Rightarrow y = \frac{4.0\times10^{-3}\text{ m}}{3} = 1.3 \text{ mm}$$

22.34. Solve: According to Equation 22.7, the fringe spacing between the *m* fringe and the *m* + 1 fringe is $\Delta y = \lambda L/d$. Δy can be obtained from Figure P22.34. The separation between the $m = 2$ fringes is 2.0 cm implying that the separation between the two consecutive fringes is $\frac{1}{4}$(2.0 cm) = 0.50 cm. Thus,

$$\Delta y = 0.50\times10^{-2}\ \text{m} = \frac{\lambda L}{d} \Rightarrow L = \frac{d\Delta y}{\lambda} = \frac{\left(0.20\times10^{-3}\ \text{m}\right)\left(0.50\times10^{-2}\ \text{m}\right)}{600\times10^{-9}\ \text{m}} = 167\ \text{cm}$$

Assess: A distance of 167 cm from the slits to the screen is reasonable.

22.35. Solve: According to Equation 22.7, the fringe spacing between the *m* fringe and the *m* + 1 fringe is $\Delta y = \lambda L/d$. Δy can be obtained from Figure P22.34. Because the separation between the *m* = 2 fringes is 2.0 cm, two consecutive fringes are $\Delta y = \frac{1}{4}(2.0\text{ cm}) = 0.50\text{ cm}$ apart. Thus,

$$\Delta y = 0.50\times10^{-2}\text{ m} = \frac{\lambda L}{d} \Rightarrow \lambda = \frac{d\Delta y}{L} = \frac{(0.20\times10^{-3}\text{ m})(0.50\times10^{-2}\text{ m})}{2.0\text{ m}} = 500\text{ nm}$$

22.36. **Solve:** The intensity of light of a double-slit interference pattern at a position y on the screen is

$$I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

where I_1 is the intensity of the light from each slit alone. At the center of the screen, that is, at y = 0 m, $I_1 = \frac{1}{4} I_{\text{double}}$. From Figure P22.34, I_{double} at the central maximum is 12 mW/m^2. So, the intensity due to a single slit is $I_1 = 3$ mW/m^2.

22.37. Model: A diffraction grating produces an interference pattern.

Visualize: The interference pattern looks like the diagram in Figure 22.8.

Solve: 500 lines per mm on the diffraction grating gives a spacing between the two lines of $d = 1 \text{ mm}/500 = (1\times10^{-3} \text{ m})/500 = 2.0\times10^{-6} \text{ m}$. The wavelength diffracted at angle $\theta_m = 30°$ in order *m* is

$$\lambda = \frac{d\sin\theta_m}{m} = \frac{(2.0\times10^{-6} \text{ m})\sin 30°}{m} = \frac{1000 \text{ nm}}{m}$$

We're told it is *visible* light that is diffracted at $30°$, and the wavelength range for visible light is 400–700 nm. Only $m = 2$ gives a visible light wavelength, so $\lambda = 500$ nm.

22.38. Model: Assume the screen is centered behind the slit.

Visualize: Refer to Figure 22.7. Think carefully about the situation. The longest wavelength that will show three fringes on each side occurs when $y_3 = 0.50$ m; but the shortest wavelength that will show three fringes on each side occurs when we *almost* let the fourth fringe on the screen, *i.e.,* when $y_4 = 0.50$ m. Use Equation 22.15: $d\sin\theta_m = m\lambda$, and Equation 22.16: $y_m = L\tan\theta_m$. We are given $L = 1.0$ m, and $d = \frac{1}{200}$ mm.

Solve: Solve Equation 22.16 for θ_m and insert it in Equation 22.15.

$$\theta_m = \tan^{-1}\frac{y_m}{L}$$

Solve Equation 22.15 for λ.

$$\lambda = \frac{d}{m}\sin\theta_m = \frac{d}{m}\sin\left(\tan^{-1}\frac{y_m}{L}\right)$$

λ_{max} occurs when $y_3 = 0.50$ m; and λ_{min} occurs when $y_4 = 0.50$ m.

$$\lambda_{min} = \frac{d}{m}\sin\theta_4 = \frac{d}{m}\sin\left(\tan^{-1}\frac{y_4}{L}\right) = \frac{\frac{1}{200}\text{ mm}}{4}\sin\left(\tan^{-1}\frac{0.50\text{ m}}{1.0\text{ m}}\right) = 560\text{ nm}$$

$$\lambda_{max} = \frac{d}{m}\sin\theta_3 = \frac{d}{m}\sin\left(\tan^{-1}\frac{y_3}{L}\right) = \frac{\frac{1}{200}\text{ mm}}{3}\sin\left(\tan^{-1}\frac{0.50\text{ m}}{1.0\text{ m}}\right) = 750\text{ nm}$$

Assess: These are reasonable wavelengths, either in or close to the visible range.

22.39. Model: Each wavelength of light is diffracted at a different angle by a diffraction grating.

Solve: Light with a wavelength of 501.5 nm creates a first-order fringe at $y = 21.90$ cm. This light is diffracted at angle

$$\theta_1 = \tan^{-1}\left(\frac{21.90\text{ cm}}{50.00\text{ cm}}\right) = 23.65°$$

We can then use the diffraction equation $d\sin\theta_m = m\lambda$, with $m = 1$, to find the slit spacing:

$$d = \frac{\lambda}{\sin\theta_1} = \frac{501.5\text{ nm}}{\sin(23.65°)} = 1250\text{ nm}$$

The unknown wavelength creates a first order fringe at $y = 31.60$ cm, or at angle

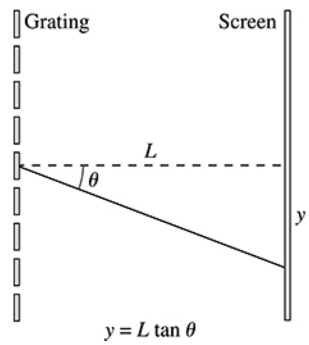
$$\theta_1 = \tan^{-1}\left(\frac{31.60\text{ cm}}{50.00\text{ cm}}\right) = 32.29°$$

With the split spacing now known, we find that the wavelength is

$$\lambda = d\sin\theta_1 = (1250\text{ nm})\sin(32.29°) = 667.8\text{ nm}$$

Assess: The distances to the fringes and the first wavelength were given to 4 significant figures. Consequently, we can determine the unknown wavelength to 4 significant figures.

22.40. Model: A diffraction grating produces an interference pattern like the diagram of Figure 22.8.
Visualize:



Solve: **(a)** A key statement is that the lines are *seen* on the screen. This means that the light is *visible* light, in the range 400 nm–700 nm. We can determine where the entire visible spectrum falls on the screen for different values of *m*. We do this by finding the angles θ_m at which 400 nm light and 700 nm light are diffracted. We then use $y_m = L\tan\theta_m$ to find their positions on the screen which is at a distance L = 75 cm. The slit spacing is $d = 1\text{ mm}/1200 = 8.333\times10^{-7}\text{ m}$. For $m = 1$,

$$\lambda = 400\text{ nm: } \theta_1 = \sin^{-1}(400\text{ nm}/d) = 28.7° \Rightarrow y_1 = 75\text{ cm } \tan 28.7° = 41\text{ cm}$$

$$\lambda = 700\text{ nm: } \theta_1 = \sin^{-1}(700\text{ nm}/d) = 57.1° \Rightarrow y_1 = 75\text{ cm } \tan 57.1° = 116\text{ cm}$$

For $m = 2$,

$$\lambda = 400\text{ nm: } \theta_1 = \sin^{-1}(2\cdot 400\text{ nm}/d) = 73.8° \Rightarrow y_2 = 75\text{ cm } \tan 73.8° = 257\text{ cm}$$

For the 700 nm wavelength at $m = 2$, $\theta_2 = \sin^{-1}(2\cdot 700\text{ nm}/d) = \sin^{-1}(1.68)$ is not defined, so $y_2 \to \infty$. We see that visible light diffracted at $m = 1$ will fall in the range 41 cm $\le y \le$ 116 and that visible light diffracted at $m = 2$ will fall in the range $y \ge$ 257 cm. These ranges do not overlap, so we can conclude with certainty that the observed diffraction lines are all $m = 1$.

(b) To determine the wavelengths, we first find the diffraction angle from the observed position by using

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{y}{75\text{ cm}}\right)$$

This angle is then used in the diffraction grating equation for the wavelength with $m = 1$, $\lambda = d\sin\theta_1$.

Line	θ	λ
56.2 cm	36.85°	500 nm
65.9 cm	41.30°	550 nm
93.5 cm	51.27°	650 nm

22.41. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram of Figure 22.8.
Solve: (a) A grating diffracts light at angles $\sin\theta_m = m\lambda/d$. The distance between adjacent slits is $d = 1\text{ mm}/600 = 1.667\times10^{-6}\text{ m} = 1667\text{ nm}$. The angle of the *m* = 1 fringe is

$$\theta_1 = \sin^{-1}\left(\frac{500\text{ nm}}{1667\text{ nm}}\right) = 17.46°$$

The distance from the central maximum to the *m* = 1 bright fringe on a screen at distance *L* is

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(Note that the small angle approximation is *not* valid for the maxima of diffraction gratings, which almost always have angles > 10°.) There are two *m* = 1 bright fringes, one on either side of the central maximum. The distance between them is $\Delta y = 2y_1 = 1.258\text{ m} \approx 1.3\text{ m}$.
(b) The maximum number of fringes is determined by the maximum value of *m* for which $\sin\theta_m$ does not exceed 1 because there are no physical angles for which $\sin\theta > 1$. In this case,

$$\sin\theta_m = \frac{m\lambda}{d} = \frac{m(500\text{ nm})}{1667\text{ nm}}$$

We can see by inspection that *m* = 1, *m* = 2, and *m* = 3 are acceptable, but *m* = 4 would require a physically impossible $\sin\theta_4 > 1$. Thus, there are three bright fringes on either side of the central maximum plus the central maximum itself for a total of seven bright fringes.

22.42. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m that are determined by Equation 22.15.

Solve: For the $m = 3$ maximum of the red light and the $m = 5$ maximum of the unknown wavelength, Equation 22.15 gives

$$d\sin\theta_3 = 3\left(660\times10^{-9}\text{ m}\right) \quad d\sin\theta_5 = 5\lambda_{\text{unknown}}$$

The $m = 5$ fringe and the $m = 3$ fringe have the same angular positions. This means $\theta_5 = \theta_3$. Dividing the two equations,

$$5\lambda_{\text{unknown}} = 3\left(660\times10^{-9}\text{ m}\right) \Rightarrow \lambda_{\text{unknown}} = 396\text{ nm}$$

22.43. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used. The visible spectrum spans the wavelengths 400 nm to 700 nm.

Solve: According to Equation 22.16, the distance y_m from the center to the mth maximum is $y_m = L\tan\theta_m$. The angle of diffraction is determined by the constructive-interference condition $d\sin\theta_m = m\lambda$, where m = 0, 1, 2, 3, … The width of the rainbow for a given fringe order is thus $w = y_{\text{red}} - y_{\text{violet}}$. The slit spacing is

$$d = \frac{1\text{ mm}}{600} = \frac{1.0\times10^{-3}\text{ m}}{600} = 1.6667\times10^{-6}\text{ m}$$

For the red wavelength and for the m = 1 order,

$$d\sin\theta_1 = (1)\lambda \Rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{d} = \sin^{-1}\frac{700\times10^{-9}\text{ m}}{1.6667\times10^{-6}\text{ m}} = 24.83°$$

From the equation for the distance of the fringe,

$$y_{\text{red}} = L\tan\theta_1 = (2.0\text{ m})\tan(24.83°) = 92.56\text{ cm}$$

Likewise for the violet wavelength,

$$\theta_1 = \sin^{-1}\left(\frac{400\times10^{-9}\text{ m}}{1.6667\times10^{-6}\text{ m}}\right) = 13.88° \Rightarrow y_{\text{violet}} = (2.0\text{ m})\tan(13.88°) = 49.42\text{ cm}$$

The width of the rainbow is thus 92.56 cm − 49.42 cm = 43.14 cm ≈ 43 cm .

22.44. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: (a) If blue light (the shortest wavelengths) is diffracted at angle θ, then red light (the longest wavelengths) is diffracted at angle $\theta + 30°$. In the first order, the equations for the blue and red wavelengths are

$$\sin\theta = \frac{\lambda_b}{d} \qquad d\sin(\theta + 30°) = \lambda_r$$

Combining the two equations we get for the red wavelength,

$$\lambda_r = d(\sin\theta\cos 30° + \cos\theta\sin 30°) = d(0.8660\sin\theta + 0.50\cos\theta) = d\left(\frac{\lambda_b}{d}\right)0.8660 + d(0.50)\sqrt{\left(1 - \frac{\lambda_b^2}{d^2}\right)}$$

$$\Rightarrow (0.50)d\sqrt{1 - \frac{\lambda_b^2}{d^2}} = \lambda_r - 0.8660\lambda_b \Rightarrow (0.50)^2(d^2 - \lambda_b^2) = (\lambda_r - 0.8660\lambda_b)^2$$

$$\Rightarrow d = \sqrt{\left(\frac{\lambda_r - 0.8660\lambda_b}{0.50}\right)^2 + \lambda_b^2}$$

Using $\lambda_b = 400 \times 10^{-9}$ m and $\lambda_r = 700 \times 10^{-9}$ m, we get $d = 8.125 \times 10^{-7}$ m. This value of d corresponds to

$$\frac{1 \text{ mm}}{d} = \frac{1.0 \times 10^{-3} \text{ m}}{8.125 \times 10^{-7} \text{ m}} = 1230 \text{ lines/mm}$$

(b) Using the value of d from part (a) and $\lambda = 589 \times 10^{-9}$ m, we can calculate the angle of diffraction as follows:

$$d\sin\theta_1 = (1)\lambda \Rightarrow (8.125 \times 10^{-7} \text{ m})\sin\theta_1 = 589 \times 10^{-9} \text{ m} \Rightarrow \theta_1 = 46.5°$$

22.45. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: An 800 line/mm diffraction grating has a slit spacing $d = (1.0\times10^{-3}\text{ m})/800 = 1.25\times10^{-6}$ m. Referring to Figure P22.45, the angle of diffraction is given by

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.436\text{ m}}{1.0\text{ m}} = 0.436 \Rightarrow \theta_1 = 23.557° \Rightarrow \sin\theta_1 = 0.400$$

Using the constructive-interference condition $d\sin\theta_m = m\lambda$,

$$\lambda = \frac{d\sin\theta_1}{1} = \left(1.25\times10^{-6}\text{ m}\right)\left(0.400\right) = 500\text{ nm}$$

We can obtain the same value of λ by using the second-order interference fringe. We first obtain θ_2:

$$\tan\theta_2 = \frac{y_2}{L} = \frac{0.436\text{ m} + 0.897\text{ m}}{1.0\text{ m}} = 1.333 \;\Rightarrow\; \theta_2 = 53.12° \;\Rightarrow\; \sin\theta_2 = 0.800$$

Using the constructive-interference condition,

$$\lambda = \frac{d\sin\theta_2}{2} = \frac{\left(1.25\times10^{-6}\text{ m}\right)\left(0.800\right)}{2} = 500\text{ nm}$$

Assess: Calculations with the first-order and second-order fringes of the interference pattern give the same value for the wavelength.

22.46. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: From Figure P22.45,

$$\tan\theta_1 = \frac{0.436 \text{ m}}{1.0 \text{ m}} = 0.436 \Rightarrow \theta_1 = 23.557° \Rightarrow \sin\theta_1 = 0.400$$

Using the constructive-interference condition $d\sin\theta_m = m\lambda$,

$$d\sin 23.557° = (1)\left(600\times10^{-9} \text{ m}\right) \Rightarrow d = \frac{600\times10^{-9} \text{ m}}{\sin(23.557°)} = 1.50\times10^{-6} \text{ m}$$

Thus, the number of lines per millimeter is

$$\frac{1.0\times10^{-3} \text{ m}}{1.50\times10^{-6} \text{ m}} = 670 \text{ lines/mm}$$

Assess: The same answer is obtained if we perform calculations using information about the second-order bright constructive-interference fringe.

22.47. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 22.14.
Solve: The dark fringes are located at

$$y_p = \frac{p\lambda L}{a} \quad p = 0, 1, 2, 3, \ldots$$

The locations of the first and third dark fringes are

$$y_1 = \frac{\lambda L}{a} \qquad y_3 = \frac{3\lambda L}{a}$$

Subtracting the two equations,

$$(y_3 - y_1) = \frac{2\lambda L}{a} \Rightarrow a = \frac{2\lambda L}{y_3 - y_1} = \frac{2(589\times10^{-9}\ \text{m})(0.75\ \text{m})}{7.5\times10^{-3}\ \text{m}} = 0.12\ \text{mm}$$

22.48. Visualize: The relationship between the diffraction grating spacing *d*, the angle at which a particular order of constructive interference occurs θ_m, the wavelength of the light, and the order of the constructive interference m is $d\sin\theta_m = m\lambda$. Also note $N = 1/d$.

Solve: The first order diffraction angle for green light is

$$\theta_1 = \sin^{-1}(\lambda/d) = \sin^{-1}(5.5\times10^{-7}\text{ m}/2.0\times10^{-6}\text{ m}) = \sin^{-1}(0.275) = 0.278\text{ rad} = 16°$$

Assess: This is a reasonable angle for a first order maximum.

22.49. Visualize: We are given $L = 1.50$ m and $m = 2$. We also know that for $\lambda = 610$ nm, $y_2 = 1.611$ m, and we seek λ for $y_2 = 1.606$ m. We will need to solve for *d* and use it to find the new λ. Use Equation 22.15: $d\sin\theta_m = m\lambda$, and Equation 22.16: $y_m = L\tan\theta_m$.

Solve:

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{m\lambda}{\sin\left(\tan^{-1}\frac{y_m}{L}\right)} = \frac{(2)(610 \text{ nm})}{\sin\left(\tan^{-1}\frac{1.611 \text{ m}}{1.50 \text{ m}}\right)} = 1.667 \text{ mm}$$

Now that we know d we can use it to find the new wavelength.

$$\lambda = \frac{d}{m}\sin\theta_m = \frac{d}{m}\sin\left(\tan^{-1}\frac{y_m}{L}\right) = \frac{1.667 \text{ mm}}{2}\sin\left(\tan^{-1}\frac{1.606 \text{ m}}{1.50 \text{ m}}\right) = 609 \text{ nm}$$

Assess: The two bright peaks are caused by specific wavelengths of light that differ by only 1 nm.

22.50. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 22.14.
Solve: These are not small angles, so we can't use equations based on the small-angle approximation. As given by Equation 22.19, the dark fringes in the pattern are located at $a\sin\theta_p = p\lambda$, where p = 1, 2, 3, … For the first minimum of the pattern, p = 1. Thus,

$$\frac{a}{\lambda} = \frac{p}{\sin\theta_p} = \frac{1}{\sin\theta_1}$$

For the three given angles the slit width to wavelength ratios are

$$(\textbf{a}):\ \left(\frac{a}{\lambda}\right)_{30°} = \frac{1}{\sin 30°} = 2, \qquad (\textbf{b}):\ \left(\frac{a}{\lambda}\right)_{60°} = \frac{1}{\sin 60°} = 1.15, \qquad (\textbf{c}):\ \left(\frac{a}{\lambda}\right)_{90°} = \frac{1}{\sin 90°} = 1$$

Assess: It is clear that the smaller the a/λ ratio, the wider the diffraction pattern. This is a conclusion that is contrary to what one might expect.

22.51. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 22.14.
Solve: $45°$ is *not* a small angle, so we can't use equations based on the small-angle approximation. As given by Equation 22.19, the dark fringes in the pattern are located at $a\sin\theta_p = p\lambda$, where p = 1, 2, 3, … For the first minimum to be at $45°$, p = 1 and the width of the slit is

$$a = \frac{p\lambda}{\sin\theta_p} = \frac{633\times10^{-9}\ \text{m}}{\sin 45°} = 895\ \text{nm}$$

22.52. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 22.14.
Solve: As given by Equation 22.19, the dark fringes in the pattern are located at $a\sin\theta_p = p\lambda$, where p = 1, 2, 3, … For the diffraction pattern to have no minima, the first minimum must be located at least at θ_1 = 90°. From the constructive-interference condition $a\sin\theta_p = p\lambda$, we have

$$a = \frac{p\lambda}{\sin\theta_p} \Rightarrow a = \frac{\lambda}{\sin 90°} = \lambda = 633 \text{ nm}$$

22.53. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The dark fringes in this diffraction pattern are given by Equation 22.21:

$$y_p = \frac{p\lambda L}{a} \qquad p = 1,\ 2,\ 3,\ \ldots$$

We note that the first minimum in the figure is 0.50 cm away from the central maximum. We are given a = 0.02 nm and $L = 1.5$ m.
Solve: Solve the above equation for λ.

$$\lambda = \frac{y_p a}{pL} = \frac{(0.50\times10^{-2}\ \text{m})(0.20\times10^{-3}\ \text{m})}{(1)(1.5\ \text{m})} = 670\ \text{nm}$$

Assess: 670 nm is in the visible range.

22.54. Model: A narrow slit produces a single-slit diffraction pattern.
Solve: The dark fringes in this diffraction pattern are given by Equation 22.21:

$$y_p = \frac{p\lambda L}{a} \quad p = 1, 2, 3, \ldots$$

We note from Figure P22.53 that the first minimum is 0.50 cm away from the central maximum. Thus,

$$L = \frac{ay_p}{p\lambda} = \frac{\left(0.15\times10^{-3}\text{ m}\right)\left(0.50\times10^{-2}\text{ m}\right)}{(1)\left(600\times10^{-9}\text{ m}\right)} = 1.3\text{ m}$$

Assess: This is a typical slit to screen separation.

22.55. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.

Solve: Within the small angle approximation, which is almost always valid for the diffraction of light, the width of the central maximum is

$$w = 2y_1 = 2.44\frac{\lambda L}{D}$$

From Figure P22.53, $w = 1.0$ cm, so

$$D = \frac{2.44\lambda L}{w} = \frac{2.44(500\times10^{-9}\text{ m})(1.0\text{ m})}{(1.0\times10^{-2}\text{ m})} = 0.12\text{ mm}$$

Assess: This is a typical size for an aperture to show diffraction.

22.56. **Model:** Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.

Solve: Within the small-angle approximation, the width of the central maximum is

$$w = 2.44\frac{\lambda L}{D}$$

Because $w = D$, we have

$$D = 2.44\frac{\lambda L}{D} \Rightarrow D = \sqrt{2.44\lambda L} \Rightarrow D = \sqrt{(2.44)(633\times10^{-9}\text{ m})(0.50\text{ m})} = 0.88\text{ mm}$$

22.57. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.

Solve: (a) Because the visible spectrum spans wavelengths from 400 nm to 700 nm, we take the average wavelength of sunlight to be 550 nm.

(b) Within the small-angle approximation, the width of the central maximum is

$$w = 2.44\frac{\lambda L}{D} \Rightarrow (1\times10^{-2}\text{ m}) = (2.44)\frac{(550\times10^{-9}\text{ m})(3\text{ m})}{D} \Rightarrow D = 4.03\times10^{-4}\text{ m} = 0.40\text{ mm}$$

22.58. Model: The antenna is a circular aperture through which the microwaves diffract.

Solve: **(a)** Within the small-angle approximation, the width of the central maximum of the diffraction pattern is $w = 2.44\lambda L/D$. The wavelength of the radiation is

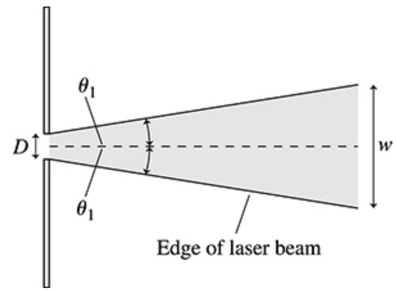
$$\lambda = \frac{c}{f} = \frac{3\times10^8 \text{ m/s}}{12\times10^9 \text{ Hz}} = 0.025 \text{ m} \Rightarrow w = \frac{2.44(0.025 \text{ m})(30\times10^3 \text{ m})}{2.0 \text{ m}} = 920 \text{ m}$$

That is, the diameter of the beam has increased from 2.0 m to 915 m, a factor of 458.

(b) The average microwave intensity is

$$I = \frac{P}{\text{area}} = \frac{100\times10^3 \text{ W}}{\pi\left[\frac{1}{2}(915 \text{ m})\right]^2} = 0.15 \text{ W/m}^2$$

22.59. Model: The laser beam is diffracted through a circular aperture.
Visualize:



Solve: **(a)** No. The laser light emerges through a circular aperture at the end of the laser. This aperture causes diffraction, hence the laser beam must gradually spread out. The diffraction angle is small enough that the laser beam *appears* to be parallel over short distances. But if you observe the laser beam at a large distance it is easy to see that the diameter of the beam is slowly increasing.
(b) The position of the first minimum in the diffraction pattern is more or less the "edge" of the laser beam. For diffraction through a circular aperture, the first minimum is at an angle

$$\theta_1 = \frac{1.22\lambda}{D} = \frac{1.22\left(633\times10^{-9}\text{ m}\right)}{0.0015\text{ m}} = 5.15\times10^{-4}\text{ rad} = 0.029°$$

(c) The diameter of the laser beam is the width of the diffraction pattern:

$$w = \frac{2.44\lambda L}{D} = \frac{2.44\left(633\times10^{-9}\text{ m}\right)(3\text{ m})}{0.0015\text{ m}} = 0.00309\text{ m} = 0.31\text{ cm}$$

(d) At L = 1 km = 1000 m, the diameter is

$$w = \frac{2.44\lambda L}{D} = \frac{2.44\left(633\times10^{-9}\text{ m}\right)(1000\text{ m})}{0.0015\text{ m}} = 1.03\text{ m} \approx 1.0\text{ m}$$

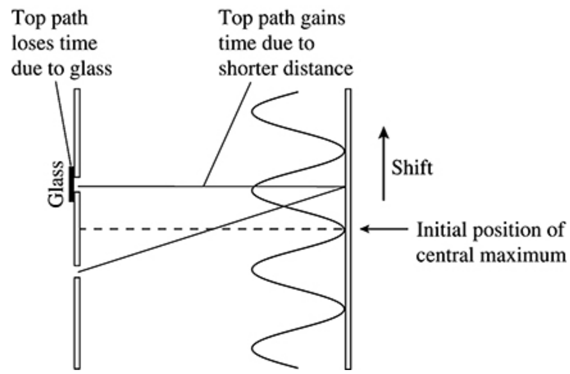
22.60. Model: The laser light is diffracted by the circular opening of the laser from which the beam emerges.
Solve: The diameter of the laser beam is the width of the central maximum. We have

$$w = \frac{2.44\lambda L}{D} \Rightarrow D = \frac{2.44\lambda L}{w} = \frac{2.44\left(532\times10^{-9}\text{ m}\right)\left(3.84\times10^{8}\text{ m}\right)}{1000\text{ m}} = 0.50\text{ m}$$

In other words, the laser beam must emerge from a laser of diameter 50 cm.

22.61. Model: Two closely spaced slits produce a double-slit interference pattern.

Visualize:



Solve: **(a)** The $m = 1$ bright fringes are separated from the $m = 0$ central maximum by

$$\Delta y = \frac{\lambda L}{d} = \frac{(600\times10^{-9}\text{ m})(1.0\text{ m})}{0.0002\text{ m}} = 0.0030\text{ m} = 3.0\text{ mm}$$

(b) The light's frequency is $f = c/\lambda = 5.00 \times 10^{14}$ Hz. Thus, the period is $T = 1/f = 2.00 \times 10^{-15}$ s. A delay of 5.0×10^{-16} s $= 0.50 \times 10^{-15}$ s is $\frac{1}{4}T$.

(c) The wave passing through the glass is delayed by $\frac{1}{4}$ of a cycle. Consequently, the two waves are not in phase as they emerge from the slits. The slits are the sources of the waves, so there is now a phase difference $\Delta\phi_0$ between the two sources. A delay of a full cycle ($\Delta t = T$) would have no effect at all on the interference because it corresponds to a phase difference $\Delta\phi_0 = 2\pi$ rad. Thus a delay of $\frac{1}{4}$ of a cycle introduces a phase difference $\Delta\phi_0 = \frac{1}{4}(2\pi) = \frac{1}{2}\pi$ rad.

(d) The text's analysis of the double-slit interference experiment assumed that the waves emerging from the two slits were in phase, with $\Delta\phi_0 = 0$ rad. Thus, there is a central maximum at a point on the screen exactly halfway between the two slits, where $\Delta\phi = 0$ rad and $\Delta r = 0$ m. Now that there is a phase difference between the sources, the central maximum—still defined as the point of constructive interference where $\Delta\phi = 0$ rad—will shift to one side. The wave leaving the slit with the glass was delayed by $\frac{1}{4}$ of a period. If it travels a *shorter* distance to the screen, taking $\frac{1}{4}$ of a period *less* than the wave coming from the other slit, it will make up for the previous delay and the two waves will arrive in phase for constructive interference. Thus, the central maximum will shift *toward* the slit with the glass. How far? A phase difference $\Delta\phi_0 = 2\pi$ would shift the fringe pattern by $\Delta y = 3.0$ mm, making the central maximum fall exactly where the $m = 1$ bright fringe had been previously. This is the point where $\Delta r = (1)\lambda$, exactly compensating for a phase shift of 2π at the slits. Thus, a phase shift of $\Delta\phi_0 = \frac{1}{2}\pi = \frac{1}{4}(2\pi)$ will shift the fringe pattern by $\frac{1}{4}(3\text{ mm}) = 0.75\text{ mm}$. The net effect of placing the glass in the slit is that the central maximum (and the entire fringe pattern) will shift 0.75 mm toward the slit with the glass.

22.62. Model: A diffraction grating produces an interference pattern, which looks like the diagram of Figure 22.8.

Solve: (a) Nothing has changed while the aquarium is empty. The order of a bright (constructive interference) fringe is related to the diffraction angle θ_m by $d\sin\theta_m = m\lambda$, where m = 0, 1, 2, 3, … The space between the slits is

$$d = \frac{1.0 \text{ mm}}{600} = 1.6667\times10^{-6} \text{ m}$$

For m = 1,

$$\sin\theta_1 = \frac{\lambda}{d} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{633\times10^{-9} \text{ m}}{1.6667\times10^{-6} \text{ m}}\right) = 22.3°$$

(b) The path-difference between the waves that leads to constructive interference is an integral multiple of the wavelength in the medium in which the waves are traveling, that is, water. Thus,

$$\lambda = \frac{633 \text{ nm}}{n_{\text{water}}} = \frac{633 \text{ nm}}{1.33} = 4.759\times10^{-7} \text{ m} \Rightarrow \sin\theta_1 = \frac{\lambda}{d} = \frac{4.759\times10^{-7} \text{ m}}{1.6667\times10^{-6} \text{ m}} = 0.2855 \Rightarrow \theta_1 = 16.6°$$

22.63. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 22.20.
Solve: The path-length difference between the two waves is $\Delta r = 2L_2 - 2L_1$. The condition for constructive interference is $\Delta r = m\lambda$, hence constructive interference occurs when

$$2(L_2 - L_1) = m\lambda \Rightarrow L_2 - L_1 = \tfrac{1}{2}m\lambda_2 = 1200\left(\tfrac{1}{2}\lambda\right) = 600\lambda$$

where λ = 632.8 nm is the wavelength of the helium-neon laser. When the mirror M_2 is moved back and a hydrogen discharge lamp is used, 1200 fringes shift again. Thus,

$$L_2' - L_1 = 1200\left(\tfrac{1}{2}\lambda'\right) = 600\lambda'$$

where $\lambda' = 656.5$ nm. Subtracting the two equations,

$$(L_2 - L_1) - (L_2' - L_1) = 600(\lambda - \lambda') = 600\left(632.8\times10^{-9}\text{ m} - 656.5\times10^{-9}\text{ m}\right)$$

$$\Rightarrow L_2' = L_2 + 14.2\times10^{-6}\text{ m}$$

That is, M_2 is now 14.2 μm closer to the beam splitter.

22.64. Model: The gas increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift.

Solve: From Equation 22.36, the number of fringe shifts is

$$\Delta m = m_2 - m_1 = (n-1)\frac{2d}{\lambda_{\text{vac}}} = (1.00028-1)\frac{(2)(2.00\times10^{-2}\ \text{m})}{600\times10^{-9}\ \text{m}} = 19$$

22.65. Model: The water increases the number of wavelengths in one arm of the interferometer.

Solve: **(a)** The incident light has λ_{vac} = 500 nm and $f = c/\lambda_{vac} = 6.00\times10^{14}$ Hz. Water doesn't affect the frequency, which is still $f_w = f = 6.00\times10^{14}$ Hz . The wavelength changes to $\lambda_w = \lambda_{vac}/n_w = 376$ nm.

(b) Light travels in a layer of thickness L twice—once to the right and then, after reflecting, once to the left—for a total distance $2L$. The *number* of wavelengths in distance $2L$ is $N = 2L/\lambda$. If the 1 mm layer is a vacuum, the number of wavelengths is

$$N_{vac} = \frac{2L}{\lambda_{vac}} = \frac{2(0.001\text{ m})}{500\times10^{-9}\text{ m}} = 4000$$

If the vacuum is replaced by 1 mm of water, the number of wavelengths is

$$N_w = \frac{2L}{\lambda_w} = \frac{2(0.001\text{ m})}{376\times10^{-9}\text{ m}} = 5319$$

So with the water added, the light travels ≈ 1320 *extra* wavelengths.

(c) Each additional wavelength of travel shifts the pattern by 1 fringe. So the addition of the water shifts the interference pattern by ≈ 1320 fringes.

22.66. Model: The piece of glass increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift.

Solve: We can rearrange Equation 22.36 to find that the index of refraction of glass is

$$n = 1 + \frac{\lambda_{vac}\Delta m}{2d} = 1 + \frac{(500\times10^{-9}\ \text{nm})(200)}{2(0.10\times10^{-3}\ \text{m})} = 1.50$$

22.67. Model: The arms of the interferometer are of equal length, so without the crystal the output would be bright.

Visualize: We need to consider how many more wavelengths fit in the electro-optic crystal than would have occupied that space (6.70 μm) without the crystal; if it is an integer then the interferometer will produce a bright output; if it is a half-integer then the interferometer will produce a dark output. But the wavelength we need to consider is the wavelength inside the crystal, not the wavelength in air.

$$\lambda_n = \frac{\lambda}{n}$$

We are told the initial *n* with no applied voltage is 1.522, and the wavelength in air is $\lambda = 1.000\ \mu$m.

Solve: The number of wavelengths that would have been in that space without the crystal is

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}} = 6.70$$

(a) With the crystal in place (and *n* = 1.522) the number of wavelengths in the crystal is

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}/1.522} = 10.20$$

$$10.20 - 6.70 = 3.50$$

which shows there are a half-integer number more wavelengths with the crystal in place than if it weren't there. Consequently the output is dark with the crystal in place but no applied voltage.

(b) Since the output was dark in the previous part, we want it to be bright in the new case with the voltage on. That means we want to have just one half more extra wavelengths in the crystal (than if it weren't there) than we did in the previous part. That is, we want 4.00 extra wavelengths in the crystal instead of 3.5, so we want 6.70 + 4.00 = 10.70 wavelengths in the crystal.

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}/n} = 10.70 \qquad \Rightarrow \qquad n = \frac{10.70(1.000\ \mu\text{m})}{6.70\ \mu\text{m}} = 1.597$$

Assess: It seems reasonable to be able to change the index of refraction of a crystal from 1.522 to 1.597 by applying a voltage.

22.68. Model: A diffraction grating produces an interference pattern like the one shown in Figure 22.8. We also assume that the small-angle approximation is valid for this grating.

Solve: (a) The general condition for constructive-interference fringes is

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

When this happens, we say that the light is diffracted at an angle θ_m. Since it is usually easier to measure distances rather than angles, we will consider the distance y_m from the center to the mth maximum. This distance is $y_m = L\tan\theta_m$. In the small-angle approximation, $\sin\theta_m \approx \tan\theta_m$, so we can write the condition for constructive interference as

$$d\frac{y_m}{L} = m\lambda \Rightarrow y_m = \frac{m\lambda L}{d}$$

The fringe separation is

$$y_{m+1} - y_m = \Delta y = \frac{\lambda L}{d}$$

(b) Now the laser light falls on a film that has a series of "slits" (*i.e.*, bright and dark stripes), with spacing

$$d' = \frac{\lambda L}{d}$$

Applying once again the condition for constructive interference:

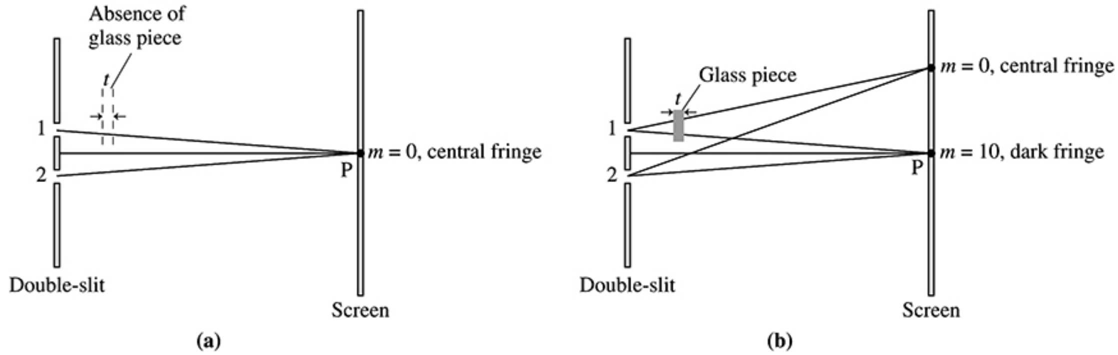
$$d'\sin\theta_m = m\lambda \;\Rightarrow\; d'\frac{y'_m}{L} = m\lambda \;\Rightarrow\; y'_m = \frac{m\lambda L}{d'} = \frac{m\lambda L}{\lambda L/d} = md$$

The fringe separation is $y'_{m+1} - y'_m = \Delta y' = d$.

That is, using the film as a diffraction grating produces a diffraction pattern whose fringe spacing is d, the spacing of the original slits.

22.69. Model: Two closely spaced slits produce a double-slit interference pattern. The interference pattern is symmetrical on both sides of the central maximum.

Visualize:



In figure (a), the interference of laser light from the two slits forms a constructive-interference central ($m = 0$) fringe at P. The paths 1P and 2P are equal. When a glass piece of thickness t is inserted in the path 1P, the interference between the two waves yields the 10th dark fringe at P. Note that the glass piece is not present in figure (a).

Solve: The number of wavelengths in the air-segment of thickness t is

$$m_1 = \frac{t}{\lambda}$$

The number of wavelengths in the glass piece of thickness t is

$$m_2 = \frac{t}{\lambda_{\text{glass}}} = \frac{t}{\lambda/n} = \frac{nt}{\lambda}$$

The path length has thus *increased* by Δm wavelengths, where

$$\Delta m = m_2 - m_1 = (n-1)\frac{t}{\lambda}$$

From the 10th dark fringe to the 1st dark fringe is 9 fringes and from the 1st dark fringe to the 0th bright fringe is one-half of a fringe. Hence,

$$\Delta m = 9 + \frac{1}{2} = \frac{19}{2} \Rightarrow \frac{19}{2} = (n-1)\frac{t}{\lambda} \Rightarrow t = \frac{19}{2}\frac{\lambda}{(n-1)} = \frac{19}{2}\frac{(633\times10^{-9}\text{ m})}{1.5-1.0} = 12.0\ \mu\text{m}$$

22.70. Visualize: To find the location y where the intensity is I_1 use Equation 22.14: $I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$. Then divide by the distance to the first minimum $y_0 = \frac{1}{2}\frac{\lambda L}{d}$ to get the fraction desired.

Solve: First set $I_{\text{double}} = I_1$.

$$I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

$$I_1 = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

$$\frac{1}{4} = \cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

$$\frac{1}{2} = \cos\left(\frac{\pi d}{\lambda L}y\right)$$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{1}{2}\right)$$

Now set up the ratio that will give the desired fraction.

$$\frac{y}{y_0'} = \frac{\frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{1}{2}\right)}{\frac{1}{2}\frac{\lambda L}{d}} = \frac{2}{\pi}\cos^{-1}\left(\frac{1}{2}\right) = \frac{2}{\pi}\frac{\pi}{3} = \frac{2}{3}$$

Assess: The fraction must be less than 1, and $\frac{2}{3}$ seems reasonable.

22.71. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m that are determined by Equation 22.15.

Solve: (a) The condition for bright fringes is $d\sin\theta = m\lambda$. If λ changes by a very small amount $\Delta\lambda$, such that θ changes by $\Delta\theta$, then we can approximate $\Delta\lambda/\Delta\theta$ as the derivative $d\lambda/d\theta$:

$$\lambda = \frac{d}{m}\sin\theta \Rightarrow \frac{\Delta\lambda}{\Delta\theta} \approx \frac{d\lambda}{d\theta} = \frac{d}{m}\cos\theta = \frac{d}{m}\sqrt{1-\sin^2\theta} = \frac{d}{m}\sqrt{1-\left(\frac{m\lambda}{d}\right)^2} = \sqrt{\left(\frac{d}{m}\right)^2 - \lambda^2}$$

$$\Rightarrow \Delta\theta = \frac{\Delta\lambda}{\sqrt{\left(\frac{d}{m}\right)^2 - \lambda^2}}$$

(b) We can now obtain the first-order and second-order angular separations for the wavelengths λ = 589.0 nm and $\lambda + \Delta\lambda$ = 589.6 nm. The slit spacing is

$$d = \frac{1.0\times10^{-3}\text{ m}}{600} = 1.6667\times10^{-6}\text{ m}$$

The first-order (m = 1) angular separation is

$$\Delta\theta = \frac{0.6\times10^{-9}\text{ m}}{\sqrt{2.7778\times10^{-12}\text{ m}^2 - 0.3476\times10^{-12}\text{ m}^2}} = \frac{0.6\times10^{-9}\text{ m}}{1.5589\times10^{-6}\text{ m}}$$
$$= 3.85\times10^{-4}\text{ rad} = 0.022°$$

The second order (m = 2) angular separation is

$$\Delta\theta = \frac{0.6\times10^{-9}\text{ m}}{\sqrt{0.6945\times10^{-12}\text{ m}^2 - 0.3476\times10^{-12}\text{ m}^2}} = 1.02\times10^{-3}\text{ rad} = 0.058°$$

22.72. Model: The intensity in a double-slit interference pattern is determined by diffraction effects from the slits.

Solve: **(a)** For the two wavelengths λ and $\lambda + \Delta\lambda$ passing simultaneously through the grating, their first-order peaks are at

$$y_1 = \frac{\lambda L}{d} \qquad y_1' = \frac{(\lambda + \Delta\lambda)L}{d}$$

Subtracting the two equations gives an expression for the separation of the peaks:

$$\Delta y = y_1' - y_1 = \frac{\Delta\lambda L}{d}$$

(b) For a double-slit, the intensity pattern is

$$I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

The intensity oscillates between zero and $4I_1$, so the maximum intensity is $4I_1$. The width is measured at the point where the intensity is half of its maximum value. For the intensity to be $\frac{1}{2}I_{\text{max}} = 2I_1$ for the $m = 1$ peak:

$$2I_1 = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y_{\text{half}}\right) \Rightarrow \cos^2\left(\frac{\pi d}{\lambda L} y_{\text{half}}\right) = \frac{1}{2} \Rightarrow \frac{\pi d}{\lambda L} y_{\text{half}} = \frac{\pi}{4} \Rightarrow y_{\text{half}} = \frac{\lambda L}{4d}$$

The width of the fringe is twice y_{half}. This means

$$w = 2y_{\text{half}} = \frac{\lambda L}{2d}$$

But the location of the $m = 1$ peak is $y_1 = \lambda L/d$, so we get $w = \frac{1}{2} y_1$.

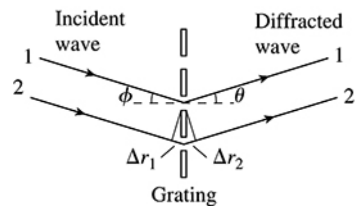
(c) We can extend the result obtained in (b) for two slits to $w = y_1/N$. The condition for barely resolving two diffraction fringes or peaks is $w = \Delta y_{\text{min}}$. From part (a) we have an expression for the separation of the first-order peaks and from part (b) we have an expression for the width. Thus, combining these two pieces of information,

$$\frac{y_1}{N} = \Delta y_{\text{min}} = \frac{\Delta\lambda_{\text{min}} L}{d} \Rightarrow \Delta\lambda_{\text{min}} = \frac{y_1 d}{LN} = \left(\frac{\lambda L}{d}\right)\frac{d}{NL} = \frac{\lambda}{N}$$

(d) Using the result of part (c),

$$N = \frac{\lambda}{\Delta\lambda_{\text{min}}} = \frac{656.27 \times 10^{-9}\ \text{m}}{0.18 \times 10^{-9}\ \text{m}} = 3646 \text{ lines}$$

22.73. Model: A diffraction grating produces an interference pattern like the one shown in Figure 22.8, when the incident light is normal to the grating. The equation for constructive interference will change when the light is incident at a nonzero angle.



Solve: (a) The path difference between waves 1 and 2 on the incident side is $\Delta r_1 = d\sin\phi$. The path difference between the waves 1 and 2 on the diffracted side, however, is $\Delta r_2 = d\sin\theta$. The total path difference between waves 1 and 2 is thus $\Delta r_1 + \Delta r_2 = d(\sin\theta + \sin\phi)$. Because the path difference for constructive interference must be equal to $m\lambda$,

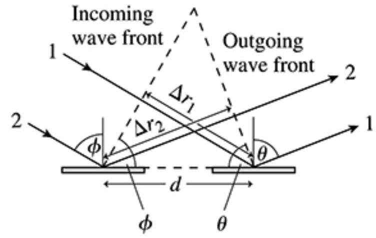
$$d(\sin\theta + \sin\phi) = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

(b) For $\phi = 30°$ the angles of diffraction are

$$\sin\theta_1 = \frac{(1)(500\times10^{-9}\ \text{m})}{(1.0\times10^{-3}\ \text{m})/600} - \sin 30° = -0.20 \Rightarrow \theta_1 = -11.5°$$

$$\sin\theta_{-1} = \frac{(-1)(500\times10^{-9}\ \text{m})}{(1.0\times10^{-3}\ \text{m})/600} - \sin 30° = -0.80 \Rightarrow \theta_{-1} = -53.1°$$

22.74. Solve: (a)



We have two incoming and two diffracted light rays at angles ϕ and θ and two wave fronts perpendicular to the rays. We can see from the figure that the wave 1 travels an extra distance $\Delta r = d\sin\phi$ to reach the reflection spot. Wave 2 travels an extra distance $\Delta r = d\sin\theta$ from the reflection spot to the outgoing wave front. The path *difference* between the two waves is

$$\Delta r = \Delta r_1 - \Delta r_2 = d(\sin\theta - \sin\phi)$$

(b) The condition for diffraction, with all the waves in phase, is still $\Delta r = m\lambda$. Using the results from part (a), the diffraction condition is

$$d\sin\theta_m = m\lambda + d\sin\phi \quad m = \ldots -2, -1, 0, 1, 2, \ldots$$

Negative values of m will give a different diffraction angle than the corresponding positive values.

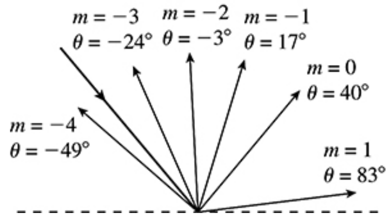
(c) The "zeroth order" diffraction from the reflection grating is $m = 0$. From the diffraction condition of part (b), this implies $d\sin\theta_0 = d\sin\phi$ and hence $\theta_0 = \phi$. That is, the zeroth order diffraction obeys the *law of reflection*—the angle of reflection equals the angle of incidence.

(d) A 700 lines per millimeter grating has spacing $d = \frac{1}{700}\ \text{mm} = 1.429\times10^{-6}\ \text{m}$ = 1429 nm. The diffraction angles are given by

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d} + \sin\phi\right) = \sin^{-1}\left(\frac{m(500\ \text{nm})}{1429\ \text{nm}} + \sin 40°\right)$$

M	θ_m
≤−5	not defined
−4	−49.2°
−3	−24.0°
−2	−3.3°
−1	17.0°
0	40.0°
1	83.1°
≥ 2	not defined

(e)



22.75. Model: Diffraction patterns from two objects can just barely be resolved if the central maximum of one image falls on the first dark fringe of the other image.
Solve: **(a)** Using Equation 22.24 with the width equal to the aperture diameter,

$$w = D = \frac{2.44\lambda L}{D} \Rightarrow D = \sqrt{2.44\lambda L} = \sqrt{(2.44)\left(550\times10^{-7}\text{ m}\right)(0.20\text{ m})} = 0.52\text{ mm}$$

(b) We can now use Equation 22.23 to find the angle between two distant sources that can be resolved. The angle is

$$\alpha = 1.22\frac{\lambda}{D} = \frac{1.22\left(550\times10^{-9}\text{ m}\right)}{0.52\times10^{-3}\text{ m}} = 1.29\times10^{-3}\text{ rad} = 0.074°$$

(c) The distance that can be resolved is

$$(1000\text{ m})\alpha = (1000\text{ m})\left(1.29\times10^{-3}\text{ rad}\right) = 1.3\text{ m}$$