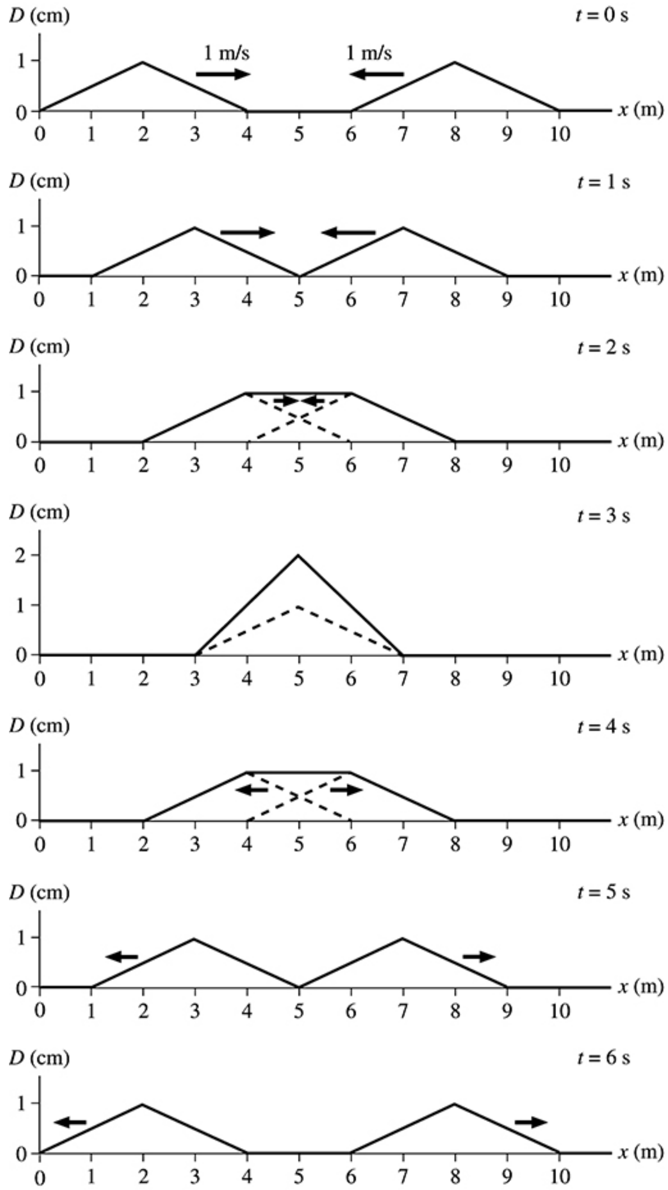
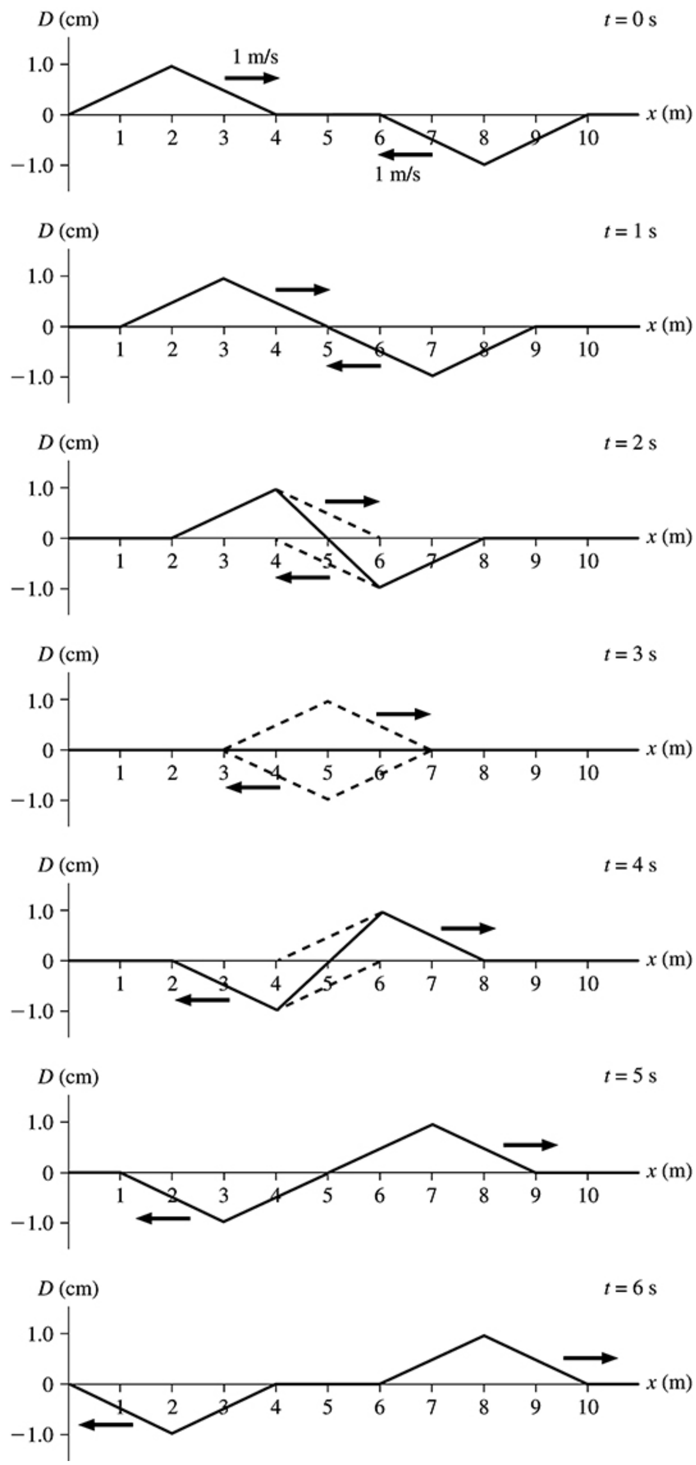
21.1. Model: The principle of superposition comes into play whenever the waves overlap.
Visualize:



The graph at t = 1.0 s differs from the graph at t = 0.0 s in that the left wave has moved to the right by 1.0 m and the right wave has moved to the left by 1.0 m. This is because the distance covered by the wave pulse in 1.0 s is 1.0 m. The snapshot graphs at t = 2.0 s, 3.0 s, and 4.0 s are a superposition of the left and the right moving waves. The overlapping parts of the two waves are shown by the dotted lines.

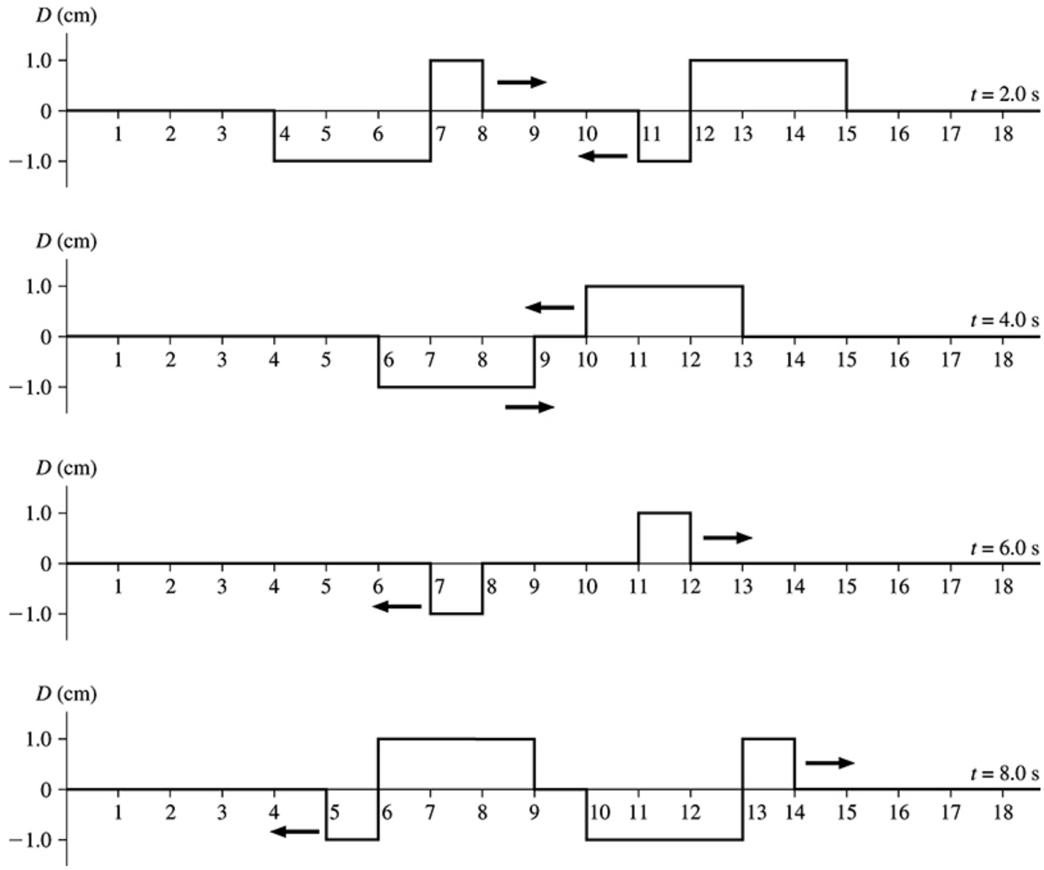
21.2. Model: The principle of superposition comes into play whenever the waves overlap.

Visualize:



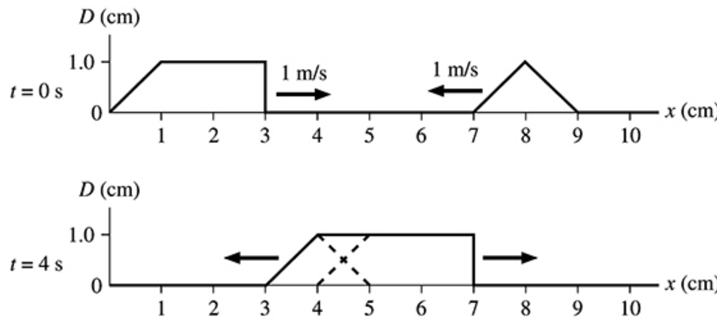
The snapshot graph at t = 1.0 s differs from the graph t = 0.0 s in that the left wave has moved to the right by 1.0 m and the right wave has moved to the left by 1.0 m. This is because the distance covered by each wave in 1.0 s is 1.0 m. The snapshot graphs at t = 2.0 s, 3.0 s, and 4.0 s are a superposition of the left and the right moving waves. The overlapping parts of the two waves are shown by the dotted lines.

21.3. Model: The principle of superposition comes into play whenever the waves overlap.
Visualize:

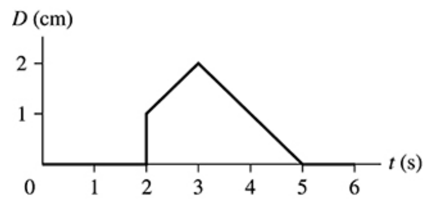


At $t = 4.0$ s the shorter pulses overlap and cancel. At $t = 6.0$ s the longer pulses overlap and cancel.

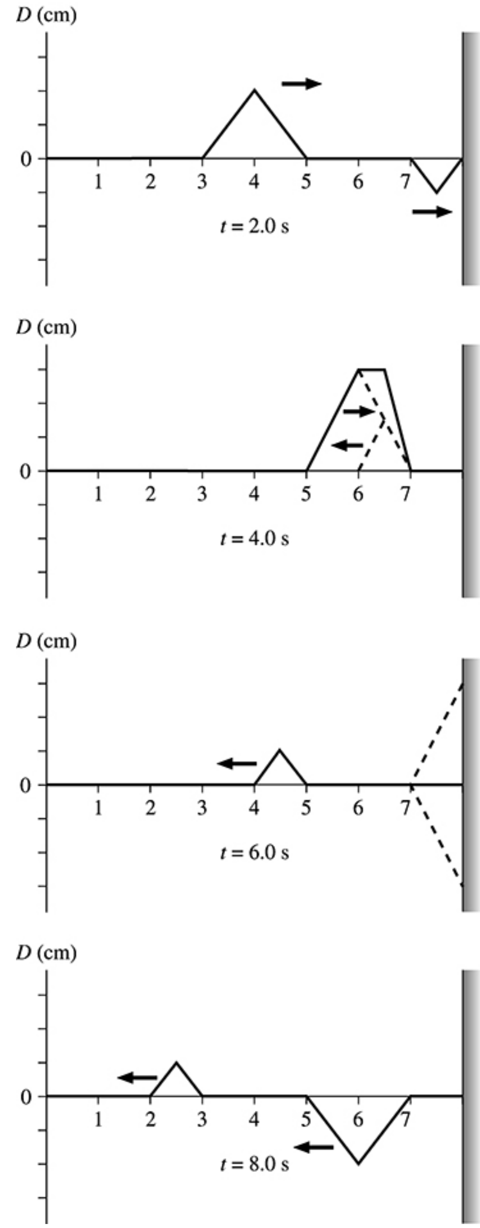
21.4. Model: The principle of superposition comes into play whenever the waves overlap.
Solve: **(a)** As graphically illustrated in the figure below, the snapshot graph of Figure EX21.5b was taken at t = 4 s.



(b)



21.5. Model: A wave pulse reflected from the string-wall boundary is inverted and its amplitude is unchanged.
Visualize:



The graph at $t = 2$ s differs from the graph at $t = 0$ s in that both waves have moved to the right by 2 m. This is because the distance covered by the wave pulse in 2 s is 2 m. The shorter pulse wave encounters the boundary wall at 2.0 s and is inverted on reflection. This reflected pulse wave overlaps with the broader pulse wave, as shown in the snapshot graph at $t = 4$ s. At $t = 6$ s, only half of the broad pulse is reflected and hence inverted; the shorter pulse wave continues to move to the left with a speed of 1 m/s. Finally, at $t = 8$ s both the reflected pulse waves are inverted and they are both moving to the left.

21.6. Model: Reflections at both ends of the string cause the formation of a standing wave.

Solve: Figure EX21.6 indicates 5/2 wavelengths on the 2.0-m-long string. Thus, the wavelength of the standing wave is $\lambda = \frac{2}{5}(2.0\text{ m}) = 0.80\text{ m}$. The frequency of the standing wave is

$$f = \frac{v}{\lambda} = \frac{40\text{ m/s}}{0.80\text{ m}} = 50\text{ Hz}$$

21.7. Model: Reflections at the string boundaries cause a standing wave on the string.

Solve: Figure EX21.7 indicates two full wavelengths on the string. Hence $\lambda = \frac{1}{2}(60\text{ cm}) = 30\text{ cm} = 0.30\text{ m}$. Thus

$$v = \lambda f = (0.30\text{ m})(100\text{ Hz}) = 30\text{ m/s}$$

21.8. Model: Reflections at the string boundaries cause a standing wave on the string.

Solve: (a) When the frequency is doubled $(f' = 2f_0)$, the wavelength is halved $(\lambda' = \frac{1}{2}\lambda_0)$. This halving of the wavelength will increase the number of antinodes to six.

(b) Increasing the tension by a factor of 4 means

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow v' = \sqrt{\frac{T'}{\mu}} = \sqrt{\frac{4T}{\mu}} = 2v$$

For the string to continue to oscillate as a standing wave with three antinodes means $\lambda' = \lambda_0$. Hence,

$$v' = 2v \Rightarrow f'\lambda' = 2f_0\lambda_0 \Rightarrow f'\lambda_0 = 2f_0\lambda_0 \Rightarrow f' = 2f_0$$

That is, the new frequency is twice the original frequency.

21.9. Model: A string fixed at both ends supports standing waves.

Solve: **(a)** We have $f_a = 24\text{ Hz} = mf_1$ where f_1 is the fundamental frequency that corresponds to $m = 1$. The next successive frequency is $f_b = 36\text{ Hz} = (m+1)f_1$. Thus,

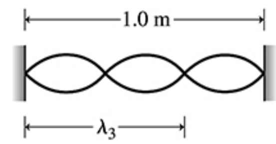
$$\frac{f_b}{f_a} = \frac{(m+1)f_1}{mf_1} = \frac{m+1}{m} = \frac{36\text{ Hz}}{24\text{ Hz}} = 1.5 \Rightarrow m+1 = 1.5m \Rightarrow m = 2 \Rightarrow f_1 = \frac{24\text{ Hz}}{2} = 12\text{ Hz}$$

The wave speed is

$$v = \lambda_1 f_1 = \frac{2L}{1} f_1 = (2.0\text{ m})(12\text{ Hz}) = 24\text{ m/s}$$

(b) The frequency of the third harmonic is 36 Hz. For $m = 3$, the wavelength is

$$\lambda_m = \frac{2L}{m} = \frac{2(1\text{ m})}{3} = \frac{2}{3}\text{ m}$$



21.10. Model: A string fixed at both ends supports standing waves.

Solve: **(a)** A standing wave can exist on the string only if its wavelength is

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, \ldots$$

The three longest wavelengths for standing waves will therefore correspond to *m* = 1, 2, and 3. Thus,

$$\lambda_1 = \frac{2(2.40 \text{ m})}{1} = 4.80 \text{ m} \qquad \lambda_2 = \frac{2(2.40 \text{ m})}{2} = 2.40 \text{ m} \qquad \lambda_3 = \frac{2(2.40 \text{ m})}{3} = 1.60 \text{ m}$$

(b) Because the wave speed on the string is unchanged from one *m* value to the other,

$$f_2\lambda_2 = f_3\lambda_3 \Rightarrow f_3 = \frac{f_2\lambda_2}{\lambda_3} = \frac{(50 \text{ Hz})(2.40 \text{ m})}{1.60 \text{ m}} = 75 \text{ Hz}$$

21.11. Model: A string fixed at both ends forms standing waves.

Solve: **(a)** The wavelength of the third harmonic is calculated as follows:

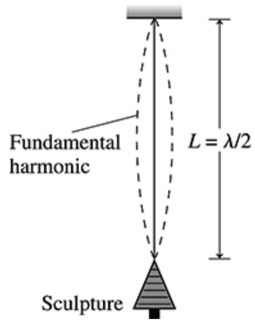
$$\lambda_m = \frac{2L}{m} \Rightarrow \lambda_3 = \frac{2L}{3} = \frac{2.42 \text{ m}}{3} = 0.807 \text{ m} \approx 0.81 \text{ m}$$

(b) The speed of waves on the string is $v = \lambda_3 f_3 = (0.807 \text{ m})(180 \text{ Hz}) = 145.3 \text{ m/s}$. The speed is also given by $v = \sqrt{T_S / \mu}$, so the tension is

$$T_S = \mu v^2 = \frac{m}{L} v^2 = \frac{0.004 \text{ kg}}{1.21 \text{ m}} (145.3 \text{ m/s})^2 = 69.7 \text{ N} \approx 70 \text{ m}$$

21.12. Model: For the stretched wire vibrating at its fundamental frequency, the wavelength of the standing wave is $\lambda_1 = 2L$.

Visualize:



Solve: The wave speed on the steel wire is

$$v_{\text{wire}} = f\lambda = f(2L) = (80\text{ Hz})(2\times 0.90\text{ m}) = 144\text{ m/s}$$

and is also equal to $\sqrt{T_S/\mu}$, where

$$\mu = \frac{\text{mass}}{\text{length}} = \frac{5.0\times 10^{-3}\text{ kg}}{0.90\text{ m}} = 5.555\times 10^{-3}\text{ kg/m}$$

The tension T_S in the wire equals the weight of the sculpture or Mg. Thus,

$$v_{\text{wire}} = \sqrt{\frac{Mg}{\mu}} \Rightarrow M = \frac{\mu v_{\text{wire}}^2}{g} = \frac{(5.555\times 10^{-3}\text{ kg/m})(144\text{ m/s})^2}{9.8\text{ m/s}^2} = 12\text{ kg}$$

21.13. Model: The laser light forms a standing wave inside the cavity.
Solve: The wavelength of the laser beam is

$$\lambda_m = \frac{2L}{m} \Rightarrow \lambda_{100,000} = \frac{2(0.5300 \text{ m})}{100,000} = 10.60\ \mu\text{m}$$

The frequency is

$$f_{100,000} = \frac{c}{\lambda_{100,000}} = \frac{3.000 \times 10^8 \text{ m/s}}{10.60 \times 10^{-6} \text{ m}} = 2.830 \times 10^{13} \text{ Hz}$$

21.14. Solve: (a) For the open-open tube, the two open ends exhibit antinodes of a standing wave. The possible wavelengths for this case are

$$\lambda_m = \frac{2L}{m} \qquad m = 1,\ 2,\ 3,\ \ldots$$

The three longest wavelengths are

$$\lambda_1 = \frac{2(1.21\ \text{m})}{1} = 2.42\ \text{m} \qquad \lambda_2 = \frac{2(1.21\ \text{m})}{2} = 1.21\ \text{m} \qquad \lambda_3 = \frac{2(1.21\ \text{m})}{3} = 0.807\ \text{m}$$

(b) In the case of an open-closed tube,

$$\lambda_m = \frac{4L}{m} \qquad m = 1,\ 3,\ 5,\ \ldots$$

The three longest wavelengths are

$$\lambda_1 = \frac{4(1.21\ \text{m})}{1} = 4.84\ \text{m} \qquad \lambda_2 = \frac{4(1.21\ \text{m})}{3} = 1.61\ \text{m} \qquad \lambda_3 = \frac{4(1.21\ \text{m})}{5} = 0.968\ \text{m}$$

21.15. Model: We have an open-open tube that forms standing sound waves.

Solve: The gas molecules at the ends of the tube exhibit maximum displacement, making antinodes at the ends. There is another antinode in the middle of the tube. Thus, this is the $m = 2$ mode and the wavelength of the standing wave is equal to the length of the tube, that is, $\lambda = 0.80$ m. Since the frequency $f = 500$ Hz, the speed of sound in this case is $v = f\lambda = (500 \text{ Hz})(0.80 \text{ m}) = 400$ m/s.

Assess: The experiment yields a reasonable value for the speed of sound.

21.16. Solve: For the open-open tube, the fundamental frequency of the standing wave is f_1 = 1500 Hz when the tube is filled with helium gas at 0°C. Using $\lambda_m = 2L/m$,

$$f_{1\text{ helium}} = \frac{v_{\text{helium}}}{\lambda_1} = \frac{970\text{ m/s}}{2L}$$

Similarly, when the tube is filled with air,

$$f_{1\text{ air}} = \frac{v_{\text{air}}}{\lambda_1} = \frac{331\text{ m/s}}{2L} \Rightarrow \frac{f_{1\text{ air}}}{f_{1\text{ helium}}} = \frac{331\text{ m/s}}{970\text{ m/s}} \Rightarrow f_{1\text{ air}} = \left(\frac{331\text{ m/s}}{970\text{ m/s}}\right)(1500\text{ Hz}) = 512\text{ Hz}$$

Assess: Note that the length of the tube is one-half the wavelength whether the tube is filled with helium or air.

21.17. Model: An organ pipe has a "sounding" hole where compressed air is blown across the edge of the pipe. This is one end of an open-open tube with the other end at the true "end" of the pipe.

Solve: For an open-open tube, the fundamental frequency is $f_1 = 16.4$ Hz. We have

$$\lambda_1 = \frac{2L}{1} \Rightarrow L = \frac{\lambda_1}{2} = \frac{1}{2}\left(\frac{v_{\text{sound}}}{f_1}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{16.4 \text{ Hz}}\right) = 10.5 \text{ m}$$

Assess: The length of the organ pipe is ≈ 34.5 feet. That is actually somewhat of an overestimate since the antinodes of real tubes are slightly outside the tube. The actual length in a real organ is about 32 feet, and this is the tallest pipe in the so called "32 foot rank" of pipes.

21.18. Model: Reflections at the string boundaries cause a standing wave on a stretched string.

Solve: Because the vibrating section of the string is 1.9 m long, the two ends of this vibrating wire are fixed, and the string is vibrating in the fundamental harmonic. The wavelength is

$$\lambda_m = \frac{2L}{m} \Rightarrow \lambda_1 = 2L = 2(1.90\ \text{m}) = 3.80\ \text{m}$$

The wave speed along the string is $v = f_1\lambda_1$ = (27.5 Hz)(3.80 m) = 104.5 m/s. The tension in the wire can be found as follows:

$$v = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v^2 = \left(\frac{\text{mass}}{\text{length}}\right)v^2 = \left(\frac{0.400\ \text{kg}}{2.00\ \text{m}}\right)(104.5\ \text{m/s})^2 = 2180\ \text{N}$$

21.19. Model: A string fixed at both ends forms standing waves.

Solve: A simple string sounds the fundamental frequency $f_1 = v/2L$. Initially, when the string is of length L_A = 30 cm, the note has the frequency $f_{1A} = v/2L_A$. For a different length, $f_{1B} = v/2L_B$. Taking the ratio of each side of these two equations gives

$$\frac{f_{1A}}{f_{1B}} = \frac{v/2L_A}{v/2L_B} = \frac{L_B}{L_A} \Rightarrow L_B = \frac{f_{1A}}{f_{1B}} L_A$$

We know that the second frequency is desired to be f_{1B} = 523 Hz. The string length must be

$$L_B = \frac{440 \text{ Hz}}{523 \text{ Hz}}(30 \text{ cm}) = 25.2 \text{ cm}$$

The question is not how long the string must be, but where must the violinist place his finger. The full string is 30 cm long, so the violinist must place his finger 4.8 cm from the end.

Assess: A fingering distance of 4.8 cm from the end is reasonable.

21.20. Model: Interference occurs according to the difference between the phases $(\Delta\phi)$ of the two waves.

Solve: (a) A separation of 20 cm between the speakers leads to maximum intensity on the *x*-axis, but a separation of 60 cm leads to zero intensity. That is, the waves are in phase when $(\Delta x)_1 = 20\text{ cm}$ but out of phase when $(\Delta x)_2 = 60\text{ cm}$. Thus,

$$(\Delta x)_2 - (\Delta x)_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(60\text{ cm} - 20\text{ cm}) = 80\text{ cm}$$

(b) If the distance between the speakers continues to increase, the intensity will again be a maximum when the separation between the speakers that produced a maximum has increased by one wavelength. That is, when the separation between the speakers is 20 cm + 80 cm = 100 cm.

21.21. Model: The interference of two waves depends on the difference between the phases $(\Delta\phi)$ of the two waves.

Solve: **(a)** Because the speakers are in phase, $\Delta\phi_0 = 0$ rad. Let d represent the path-length difference. Using m = 0 for the smallest d and the condition for destructive interference, we get

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\left(m+\tfrac{1}{2}\right)\pi \text{ rad} \qquad m = 0, 1, 2, 3 \ldots$$

$$\Rightarrow 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = \pi \text{ rad} \;\Rightarrow 2\pi\frac{d}{\lambda} + 0 \text{ rad} = \pi \text{ rad} \Rightarrow d = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{v}{f}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{686 \text{ Hz}}\right) = 0.25 \text{ m}$$

(b) When the speakers are out of phase, $\Delta\phi_0 = \pi$. Using m = 1 for the smallest d and the condition for constructive interference, we get

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2m\pi \qquad m = 0, 1, 2, 3, \ldots$$

$$\Rightarrow 2\pi\frac{d}{\lambda} + \pi = 2\pi \;\Rightarrow d = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{v}{f}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{686 \text{ Hz}}\right) = 0.25 \text{ m}$$

21.22. Model: We assume that the speakers are identical and that they are emitting in phase.

Solve: Since you don't hear anything, the separation between the two speakers corresponds to the condition of destructive interference. With $\Delta\phi_0 = 0$ rad, Equation 21.23 becomes

$$2\pi\frac{d}{\lambda} = 2\left(m+\tfrac{1}{2}\right)\pi \text{ rad} \Rightarrow d = \left(m+\tfrac{1}{2}\right)\lambda \Rightarrow d = \frac{\lambda}{2},\ \frac{3\lambda}{2},\ \frac{5\lambda}{2}$$

Since the wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{170 \text{ Hz}} = 2.0 \text{ m}$$

three possible values for d are 1.0 m, 3.0 m, and 5.0 m.

21.23. Model: Reflection is maximized if the two reflected waves interfere constructively.
Solve: The film thickness that causes constructive interference at wavelength λ is given by Equation 21.32:

$$\lambda_C = \frac{2nd}{m} \Rightarrow d = \frac{\lambda_C m}{2n} = \frac{(600 \times 10^{-9}\text{ m})(1)}{(2)(1.39)} = 216\text{ nm}$$

where we have used $m = 1$ to calculate the thinnest film.
Assess: The film thickness is much less than the wavelength of visible light. The above formula is applicable because
$n_{air} < n_{film} < n_{glass}$.

21.24. Model: Reflection is maximized if the two reflected waves interfere constructively.

Solve: The film thickness that causes constructive interference at wavelength λ is given by Equation 21.32:

$$\lambda_C = \frac{2nd}{m} \Rightarrow d = \frac{\lambda_C m}{2n} = \frac{(500\times10^{-9}\ \text{m})(1)}{(2)(1.25)} = 200\ \text{nm}$$

where we have used $m = 1$ to calculate the thinnest film.

Assess: The film thickness is much less than the wavelength of visible light. The above formula is applicable because

$n_{air} < n_{oil} < n_{water}$.

21.25. Solve: (a) The circular wave fronts emitted by the two sources show that the two sources are in phase. This is because the wave fronts of each source have moved the same distance from their sources.

(b) Let us label the top source as 1 and the bottom source as 2. Since the sources are in phase, $\Delta\phi_0 = 0$ rad. For the point P, $r_1 = 3\lambda$ and $r_2 = 4\lambda$. Thus, $\Delta r = r_2 - r_1 = 4\lambda - 3\lambda = \lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} = \frac{2\pi(\lambda)}{\lambda} = 2\pi$$

This corresponds to constructive interference.

For the point Q, $r_1 = \frac{7}{2}\lambda$ and $r_2 = 2\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} = \frac{2\pi\left(\frac{3}{2}\lambda\right)}{\lambda} = 3\pi$$

This corresponds to destructive interference.

For the point R, $r_1 = \frac{5}{2}\lambda$ and $r_2 = \frac{7}{2}\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi(\lambda)}{\lambda} = 2\pi$$

This corresponds to constructive interference.

	r_1	r_2	Δr	C/D
P	3λ	4λ	λ	C
Q	$\frac{7}{2}\lambda$	2λ	$\frac{3}{2}\lambda$	D
R	$\frac{5}{2}\lambda$	$\frac{7}{2}\lambda$	λ	C

21.26. Solve: (a) The circular wave fronts emitted by the two sources indicate the sources are out of phase. This is because the wave fronts of each source have not moved the same distance from their sources.

(b) Let us label the top source as 1 and the bottom source as 2. The phase difference between the sources is $\Delta\phi_0 = \pi$. For the point *P*, $r_1 = 2\lambda$ and $r_2 = 3\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(3\lambda - 2\lambda)}{\lambda} + \pi = 3\pi$$

This corresponds to destructive interference.

For the point *Q*, $r_1 = 3\lambda$ and $r_2 = \frac{3}{2}\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\left(\frac{3}{2}\lambda\right)}{\lambda} + \pi = 4\pi$$

This corresponds to constructive interference.

For the point *R*, $r_1 = \frac{5}{2}\lambda$ and $r_2 = 3\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\left(\frac{1}{2}\lambda\right)}{\lambda} + \pi = 2\pi$$

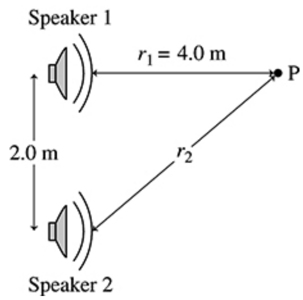
This corresponds to constructive interference.

	r_1	r_2	Δr	C/D
P	2λ	3λ	λ	D
Q	3λ	$\frac{3}{2}\lambda$	$\frac{3}{2}\lambda$	C
R	$\frac{5}{2}\lambda$	3λ	$\frac{1}{2}\lambda$	C

Assess: Note that it is not r_1 or r_2 that matter, but the difference Δr between them.

21.27. Model: The two speakers are identical, and so they are emitting circular waves in phase. The overlap of these waves causes interference.

Visualize:



Solve: From the geometry of the figure,

$$r_2 = \sqrt{r_1^2 + (2.0\text{ m})^2} = \sqrt{(4.0\text{ m})^2 + (2.0\text{ m})^2} = 4.472\text{ m}$$

So, $\Delta r = r_2 - r_1 = 4.472\text{ m} - 4.0\text{ m} = 0.472\text{ m}$. The phase difference between the sources is $\Delta\phi_0 = 0$ rad and the wavelength of the sound waves is

$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{1800\text{ Hz}} = 0.1889\text{ m}$$

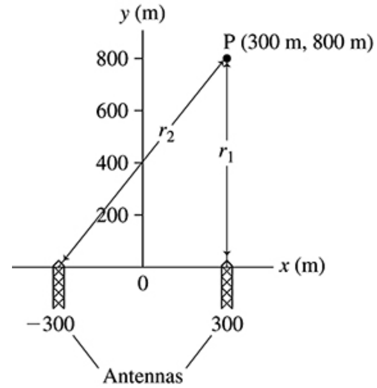
Thus, the phase difference of the waves at the point 4.0 m in front of one source is

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(0.472\text{ m})}{0.1889\text{ m}} + 0\text{ rad} = 5\pi\text{ rad} = 2.5(2\pi\text{ rad})$$

This is a half-integer multiple of 2π rad, so the interference is perfect destructive.

21.28. Model: The two radio antennas are emitting out-of-phase, circular waves. The overlap of these waves causes interference.

Visualize:



Solve: From the geometry of the figure, $r_1 = 800$ m and

$$r_2 = \sqrt{(800\text{ m})^2 + (600\text{ m})^2} = 1000\text{ m}$$

So, $\Delta r = r_2 - r_1 = 200$ m and $\Delta\phi_0 = \pi$ rad. The wavelength of the waves is

$$\lambda = \frac{c}{f} = \frac{3.0\times 10^8\text{ m/s}}{3.0\times 10^6\text{ Hz}} = 100\text{ m}$$

Thus, the phase difference of the waves at the point (300 m, 800 m) is

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(200\text{ m})}{100\text{ m}} + \pi\text{ rad} = 5\pi\text{ rad} = 2.5(2\pi\text{ rad})$$

This is a half-integer multiple of 2π rad, so the interference is perfect destructive.

21.29. Solve: The beat frequency is

$$f_{\text{beat}} = f_1 - f_2 \Rightarrow 3 \text{ Hz} = f_1 - 200 \text{ Hz} \Rightarrow f_1 = 203 \text{ Hz}$$

f_1 is larger than f_2 because the increased tension increases the wave speed and hence the frequency.

21.30. Solve: The flute player's initial frequency is either 523 Hz + 4 Hz = 527 Hz or 523 Hz − 4 Hz = 519 Hz. Since she matches the tuning fork's frequency by lengthening her flute, she is increasing the wavelength of the standing wave in the flute. A wavelength increase means a decrease of frequency because $v = f\lambda$. Thus, her initial frequency was 527 Hz.

21.31. Model: The superposition of two slightly different frequencies creates beats.

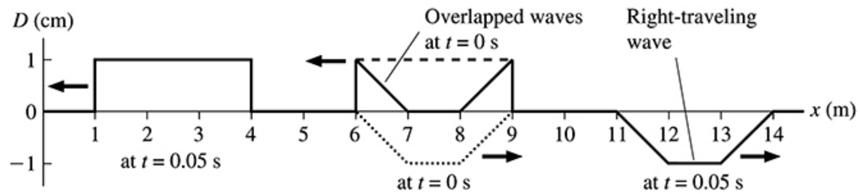
Solve: Let $\lambda_1 = 780.54510\text{ nm}$ and $\lambda_2 > \lambda_1$. This means $f_2 < f_1$ and

$$\Delta f = f_1 - f_2 = \frac{c}{\lambda_1} - \frac{c}{\lambda_2} = 98.5 \times 10^6 \text{ Hz}$$

$$\Rightarrow \lambda_2 = \frac{1}{(1/\lambda_1) - (\Delta f/c)} = \frac{1}{1/(780.54510 \times 10^{-9}\text{ m}) - (98.5 \times 10^6 \text{ Hz})/(3.00 \times 10^8 \text{ m/s})}$$
$$= 780.54530 \text{ nm}$$

Assess: A small difference in wavelengths, $(\lambda_2 - \lambda_1) = 0.00020\text{ nm} = 0.20\text{ pm}$, can yield beats at a relatively high frequency of 98.5 MHz.

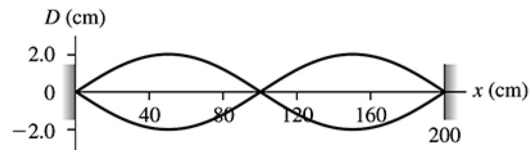
21.32. Model: The principle of superposition applies to overlapping waves.
Visualize:



Solve: Because the wave pulses travel along the string at a speed of 100 m/s, they move a distance of $d = vt = (100 \text{ m/s})(0.05 \text{ s}) = 5$ m in 0.050 s. The front of the wave pulse moving left, which is located at $x = 1$ m at $t = 0.050$ s, was thus located at $x = 6$ m at $t = 0$ s. This helps us draw the snapshot of the wave pulse moving left at $t = 0$ s (shown as a dashed line). Subtracting this wave snapshot from the resultant at $t = 0$ s (shown as a solid line) yields the right-traveling wave's snapshot at $t = 0$ s (shown as a dotted line). Finally, the snapshot graph of the wave pulse moving right at $t = 0.050$ s is the same as at $t = 0$ s (shown as a dotted line) except that it is shifted to the right by 5 m.

21.33. Model: The wavelength of the standing wave on a string vibrating at its second-harmonic frequency is equal to the string's length.

Visualize:



Solve: The length of the string $L = 2.0$ m, so $\lambda = L = 2.0$ m. This means the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.0 \text{ m}} = \pi \text{ rad/m}$$

According to Equation 21.5, the displacement of a medium when two sinusoidal waves superpose to give a standing wave is $D(x,t) = A(x)\cos\omega t$, where $A(x) = 2a\sin kx = A_{\text{max}}\sin kx$. The amplitude function gives the amplitude of oscillation from point to point in the medium. For $x = 10$ cm,

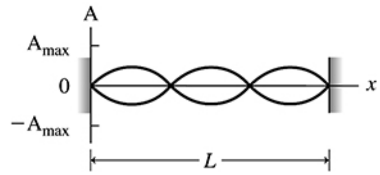
$$A(x = 10 \text{ cm}) = (2.0 \text{ cm})\sin\left[(\pi \text{ rad/m})(0.10 \text{ m})\right] = 0.62 \text{ cm}$$

Similarly, $A(x = 20 \text{ cm}) = 1.18$ cm, $A(x = 30 \text{ cm}) = 1.62$ cm, $A(x = 40 \text{ cm}) = 1.90$ cm, and $A(x = 50 \text{ cm}) = 2.00$ cm.

Assess: Consistent with the above figure, the amplitude of oscillation is a maximum at $x = 0.50$ m.

21.34. Model: The wavelength of the standing wave on a string is $\lambda_m = 2L/m$, where m = 1, 2, 3, … We assume that 30 cm is the first place from the left end of the string where $A = A_{max}/2$.

Visualize:



Solve: The amplitude of oscillation on the string is $A(x) = A_{max}\sin kx$. Since the string is vibrating in the third harmonic, the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2L/3)} = 3\frac{\pi}{L}$$

Substituting into the equation for the amplitude,

$$\frac{1}{2}A_{max} = A_{max}\sin\left(\frac{3\pi}{L}(0.30\text{ m})\right) \Rightarrow \sin\left(\frac{3\pi}{L}(0.30\text{ m})\right) = \frac{1}{2} \Rightarrow \frac{3\pi}{L}(0.30\text{ m}) = \frac{\pi}{6}\text{ rad} \Rightarrow L = 5.4\text{ m}$$

21.35. Model: The wavelength of the standing wave on a string vibrating at its fundamental frequency is equal to $2L$.

Solve: The amplitude of oscillation on the string is $A(x) = 2a\sin kx,$ where a is the amplitude of the traveling wave and the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$$

Substituting into the above equation,

$$A\left(x = \tfrac{1}{4}L\right) = 2.0 \text{ cm} = 2a\sin\left[\left(\frac{\pi}{L}\right)\left(\frac{L}{4}\right)\right] \Rightarrow 1.0 \text{ cm} = a\left(\frac{1}{\sqrt{2}}\right) \Rightarrow a = \sqrt{2} \text{ cm} = 1.4 \text{ cm}$$

21.36. Solve: You can see in Figure 21.4 that the time between two successive instants when the antinodes are at maximum height is half the period, or $\frac{1}{2}T$. Thus $T = 2(0.25\ \text{s}) = 0.50\ \text{s}$, and so

$$f = \frac{1}{T} = \frac{1}{0.50\ \text{s}} = 2.0\ \text{Hz} \Rightarrow \lambda = \frac{v}{f} = \frac{3.0\ \text{m/s}}{2.0\ \text{Hz}} = 1.5\ \text{m}$$

21.37. Model: The wave on a stretched string with both ends fixed is a standing wave. For vibration at its fundamental frequency, $\lambda = 2L$.

Solve: The wavelength of the wave reaching your ear is 39.1 cm = 0.391 m. So the frequency of the sound wave is

$$f = \frac{v_{\text{air}}}{\lambda} = \frac{344 \text{ m/s}}{0.391 \text{ m}} = 879.8 \text{ Hz}$$

This is also the frequency emitted by the wave on the string. Thus,

$$879.8 \text{ Hz} = \frac{v_{\text{string}}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T_{\text{S}}}{\mu}} = \frac{1}{\lambda}\sqrt{\frac{150 \text{ N}}{0.0006 \text{ kg/m}}} \Rightarrow \lambda = 0.568 \text{ m}$$

$$\Rightarrow L = \tfrac{1}{2}\lambda = 0.284 \text{ m} = 28.4 \text{ cm}$$

21.38. Model: The wave on a stretched string with both ends fixed is a standing wave.

Solve: We must distinguish between the sound wave in the air and the wave on the string. The listener hears a sound wave of wavelength $\lambda_{\text{sound}} = 40\text{ cm} = 0.40\text{ m}$. Thus, the frequency is

$$f = \frac{v_{\text{sound}}}{\lambda_{\text{sound}}} = \frac{343\text{ m/s}}{0.40\text{ m}} = 857.5\text{ Hz}$$

The violin string oscillates at the same frequency, because each oscillation of the string causes one oscillation of the air. But the *wavelength* of the standing wave on the string is very different because the wave speed on the string is not the same as the wave speed in air. Bowing a string produces sound at the string's fundamental frequency, so the wavelength of the string is

$$\lambda_{\text{string}} = \lambda_1 = 2L = 0.60\text{ m} \Rightarrow v_{\text{string}} = \lambda_{\text{string}} f = (0.60\text{ m})(857.5\text{ Hz}) = 514.5\text{ m/s}$$

The tension is the string is found as follows:

$$v_{\text{string}} = \sqrt{\frac{T_{\text{S}}}{\mu}} \Rightarrow T_{\text{S}} = \mu\left(v_{\text{string}}\right)^2 = (0.001\text{ kg/m})(514.5\text{ m/s})^2 = 260\text{ N}$$

21.39. Model: A string fixed at both ends forms standing waves.

Solve: **(a)** Three antinodes means the string is vibrating as the $m = 3$ standing wave. The frequency is $f_3 = 3f_1$, so the fundamental frequency is $f_1 = \frac{1}{3}(420 \text{ Hz}) = 140$ Hz. The fifth harmonic will have the frequency $f_5 = 5f_1 =$ 700 Hz.

(b) The wavelength of the fundamental mode is $\lambda_1 = 2L = 1.20$ m. The wave speed on the string is $v = \lambda_1 f_1 = (1.20 \text{ m})$

(140 Hz) = 168 m/s. Alternatively, the wavelength of the $n = 3$ mode is $\lambda_3 = \frac{1}{3}(2L) = 0.40$ m, from which $v = \lambda_3 f_3 =$

(0.40 m)(420 Hz) = 168 m/s. The wave speed on the string is given by

$$v = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v^2 = (0.0020 \text{ kg/m})(168 \text{ m/s})^2 = 56 \text{ N}$$

Assess: You must remember to use the linear density in SI units of kg/m. Also, the speed is the same for all modes, but you must use a matching λ and f to calculate the speed.

21.40. Model: Assume that the extra kilogram doesn't stretch the wire longer (so L stays the same) nor thinner (so μ stays the same). Also assume that because the wire is thin its own weight is negligible, so T_s is constant throughout the wire and is equal to Mg.

Visualize: The wire is fixed at both ends so in the second harmonic $L = \lambda$. We are given $f_2 = 200$ Hz and $f_2' = 245$ Hz and $M' = M + 1.0$ kg. Apply $v = \lambda f$ and $v = \sqrt{T_s/\mu}$.

Solve: Cancel off λ, $\sqrt{\mu}$, and $\sqrt{g}$ in turn.

$$\frac{f_2'}{f_2} = \frac{v'/\lambda}{v/\lambda} = \frac{\sqrt{T_s'/\mu}}{\sqrt{T_s/\mu}} = \frac{\sqrt{M'g}}{\sqrt{Mg}} = \frac{\sqrt{(M+1.0\text{ kg})g}}{\sqrt{Mg}} = \sqrt{\frac{M+1.0\text{ kg}}{M}}$$

$$\left(\frac{f_2'}{f_2}\right)^2 = \frac{M+1.0\text{ kg}}{M}$$

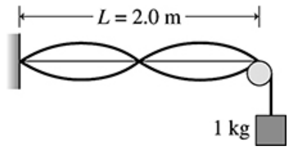
$$M\left(\left(\frac{f_2'}{f_2}\right)^2 - 1\right) = 1.0\text{ kg}$$

$$M = \frac{1.0\text{ kg}}{\left(\frac{f_2'}{f_2}\right)^2 - 1} = \frac{1.0\text{ kg}}{\left(\frac{245\text{ Hz}}{200\text{ Hz}}\right)^2 - 1} = 2.0\text{ kg}$$

Assess: We did not expect M to be really huge or a) it would have broken the wire, and b) adding one more kilogram wouldn't have made as big a difference in f_2 as it did.

21.41. Model: The stretched string with both ends fixed forms standing waves.

Visualize:



Solve: The astronauts have created a stretched string whose vibrating length is L = 2.0 m. The weight of the hanging mass creates a tension $T_S = Mg$ in the string, where M = 1.0 kg. As a consequence, the wave speed on the string is

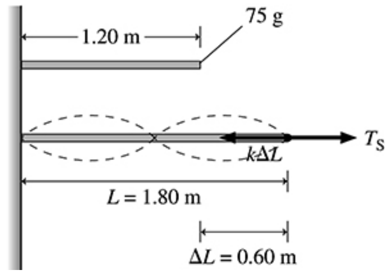
$$v = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{Mg}{\mu}}$$

where μ = (0.0050 kg)/(2.5 m) = 0.0020 kg/m is the linear density. The astronauts then observe standing waves at frequencies of 64 Hz and 80 Hz. The first is *not* the fundamental frequency of the string because 80 Hz $\neq$ 2 × 64 Hz. But we can easily show that both are multiples of 16 Hz: $64\text{ Hz} = 4f_1$ and $80\text{ Hz} = 5f_1$. Both frequencies are also multiples of 8 Hz. But 8 Hz cannot be the fundamental frequency because, if it were, there would be a standing wave resonance at 9(8 Hz) = 72 Hz. So the fundamental frequency is f_1 = 16 Hz. The fundamental wavelength is $\lambda_1 = 2L$ = 4.0 m. Thus, the wave speed on the string is $v = \lambda_1 f_1 = 64.0\text{ m/s}$. Now we can find g on Planet X:

$$v = \sqrt{\frac{Mg}{\mu}} \Rightarrow g = \frac{\mu}{M}v^2 = \frac{0.0020\text{ kg/m}}{1.0\text{ kg}}(64\text{ m/s})^2 = 8.2\text{ m/s}^2$$

21.42. Model: The stretched bungee cord that forms a standing wave with two antinodes is vibrating at the second harmonic frequency.

Visualize:



Solve: Because the vibrating cord has two antinodes, $\lambda_2 = L = 1.80$ m. The wave speed on the cord is

$$v_{\text{cord}} = f\lambda = (20 \text{ Hz})(1.80 \text{ m}) = 36 \text{ m/s}$$

The tension T_S in the cord is equal to $k\Delta L$, where k is the bungee's spring constant and ΔL is the 0.60 m the bungee has been stretched. Thus,

$$v_{\text{cord}} = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{k\Delta L}{\mu}}$$

21.43. Solve: (a) Because the frequency of the standing wave on the copper wire is the same as the frequency on the aluminum wire,

$$f_{\text{Cu}} = f_{\text{Al}} \Rightarrow \frac{v_{\text{Cu}}}{\lambda_{\text{Cu}}} = \frac{v_{\text{Al}}}{\lambda_{\text{Cu}}}$$

Let n_{Cu} be the number of half-wavelength antinodes on the copper wire and n_{Al} be the number of half-wavelength antinodes on the aluminum wire. Thus,

$$n_{\text{Cu}}\left(\frac{\lambda_{\text{Cu}}}{2}\right) = 0.22 \text{ m} \Rightarrow \lambda_{\text{Cu}} = \frac{0.44 \text{ m}}{n_{\text{Cu}}} \qquad n_{\text{Al}}\left(\frac{\lambda_{\text{Al}}}{2}\right) = 0.60 \text{ m} \Rightarrow \lambda_{\text{Al}} = \frac{1.20 \text{ m}}{n_{\text{Al}}}$$

$$\Rightarrow \frac{v_{\text{Cu}}}{(0.44 \text{ m}/n_{\text{Cu}})} = \frac{v_{\text{Al}}}{(1.20 \text{ m}/n_{\text{Al}})} \Rightarrow \frac{n_{\text{Cu}}}{n_{\text{Al}}} = \left(\frac{v_{\text{Al}}}{v_{\text{Cu}}}\right)\left(\frac{0.44 \text{ m}}{1.20 \text{ m}}\right)$$

We can find v_{Cu} and v_{Al} by using the following equations for a stretched wire:

$$v_{\text{Cu}} = \sqrt{\frac{T}{\mu_{\text{Cu}}}} \qquad v_{\text{Al}} = \sqrt{\frac{T}{\mu_{\text{Al}}}}$$

The linear densities are calculated as follows:

$$\mu_{\text{Cu}} = \frac{m_{\text{Cu}}}{0.22 \text{ m}} = \frac{\rho_{\text{Cu}} V_{\text{Cu}}}{0.22 \text{ m}} = \left(\frac{\rho_{\text{Cu}}}{0.22 \text{ m}}\right)\pi r^2 (0.22 \text{ m})$$
$$= (8920 \text{ kg/m}^3)\pi(5.0\times10^{-4} \text{ m})^2 = 7.006\times10^{-3} \text{ kg/m}^3$$

$$\mu_{\text{Al}} = \frac{m_{\text{Al}}}{0.60 \text{ m}} = \frac{\rho_{\text{Al}} V_{\text{Al}}}{0.60 \text{ m}} = \left(\frac{\rho_{\text{Al}}}{0.60 \text{ m}}\right)\pi r^2 (0.60 \text{ m})$$
$$= (2700 \text{ kg/m}^3)\pi(5.0\times10^{-4} \text{ m})^2 = 2.121\times10^{-3} \text{ kg/m}^3$$

$$\Rightarrow v_{\text{Cu}} = \sqrt{\frac{20 \text{ N}}{7.006\times10^{-3} \text{ kg/m}^3}} = 53.43 \text{ m/s} \quad v_{\text{Al}} = \sqrt{\frac{20 \text{ N}}{2.121\times10^{-3} \text{ kg/m}^3}} = 97.12 \text{ m/s}$$

Going back to the $n_{\text{Cu}}/n_{\text{Al}}$ equation, we have

$$\frac{n_{\text{Cu}}}{n_{\text{Al}}} = \left(\frac{97.12 \text{ m/s}}{53.43 \text{ m/s}}\right)\left(\frac{0.44 \text{ m}}{1.20 \text{ m}}\right) = 0.666 = \frac{2}{3} \Rightarrow n_{\text{Cu}} = 2 \text{ and } n_{\text{Al}} = 3$$

Substituting into the expressions for wavelength and frequency,

$$\lambda_{\text{Cu}} = \frac{0.44 \text{ m}}{n_{\text{Cu}}} = \frac{0.44 \text{ m}}{2} = 0.22 \text{ m} \Rightarrow f_{\text{Cu}} = \frac{v_{\text{Cu}}}{\lambda_{\text{Cu}}} = \frac{53.43 \text{ m/s}}{0.22 \text{ m}} = 243 \text{ Hz} \approx 240 \text{ Hz}$$

$$\lambda_{\text{Al}} = \frac{1.20 \text{ m}}{n_{\text{Al}}} = \frac{1.20 \text{ m}}{3} = 0.40 \text{ m} \Rightarrow f_{\text{Al}} = \frac{v_{\text{Al}}}{\lambda_{\text{Al}}} = \frac{97.12 \text{ m/s}}{0.40 \text{ m}} = 243 \text{ Hz} \approx 240 \text{ Hz}$$

(b) At this frequency of 240 Hz, there are 3 antinodes on the aluminum wire.

21.44. Visualize: Use primed quantities for when the sphere is submerged. We are given $f_5' = f_3$ and $M = 1.5\text{ kg}$. We also know the density of water is $\rho = 1000\text{ kg/m}^3$. In the third mode before the sphere is submerged $L = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2}{3}L$. Likewise, after the sphere is submerged $L = \frac{5}{2}\lambda' \Rightarrow \lambda' = \frac{2}{5}L$. The tension in the string before the sphere is submerged is $T_s = Mg$, but after the sphere is submerged, according to Archimedes' principle, it is reduced by the weight of the water displaced by the sphere: $T_s' = Mg - \rho Vg$, where $V = \frac{4}{3}\pi R^3$.

Solve: We are looking for R so solve $T_s' = Mg - \rho Vg$ for ρVg and later we will isolate R from that.

$$\rho Vg = Mg - T_s'$$

Solve $v' = \sqrt{T_s/\mu}$ for T_s'. Also substitute for V.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu v'^2$$

Now use $v' = \lambda' f'$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu(\lambda' f_5')^2$$

Recall that $f_5' = f_3$ and $\lambda' = \frac{2}{5}L$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}Lf_3\right)^2$$

Substitute $f_3 = v/\lambda$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}L\frac{v}{\lambda}\right)^2$$

Now use $\lambda = \frac{2}{3}L$ and $v = \sqrt{T_s/\mu}$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}L\frac{\sqrt{T_s/\mu}}{\frac{2}{3}L}\right)^2$$

The 2's, μ's, and L's cancel.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \left(\frac{3}{5}\sqrt{T_s}\right)^2$$

$T_s = Mg$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \left(\frac{3}{5}\right)^2 Mg$$

Cancel g and factor M out on the right side.

$$\rho\left(\frac{4}{3}\right)\pi R^3 = M\left(1-\left(\frac{3}{5}\right)^2\right) = M\left(1-\frac{9}{25}\right) = M\left(\frac{16}{25}\right)$$

Now solve for R.

$$R^3 = \left(\frac{3}{4}\right)\left(\frac{16}{25}\right)\frac{M}{\pi\rho}$$

$$R = \sqrt[3]{\frac{12}{25}\frac{M}{\pi\rho}} = \sqrt[3]{\frac{12}{25}\frac{(1.5\text{ kg})}{\pi(1000\text{ kg/m}^3)}} = 6.1\text{ cm}$$

Assess: The density of the sphere turns out to be about $1.5\times$ the density of water, which means it sinks and is in a reasonable range for densities.

21.45. Visualize: First compute the length L of the wire from the Pythagorean theorem. $L = 2\sqrt{2}$ m. Now $\mu = M/L =$ 0.075 kg/$2\sqrt{2}$ m $= 0.02652$ kg/m. Also, in the fundamental mode $\lambda = 2L$; here $\lambda = 4\sqrt{2}$ m $= 5.657$ m.

Solve: Apply the principles of statics to the point at the end of the bar.

$$\Sigma F_y = T_s \sin 45° - (8 \text{ kg})(g) = 0 \text{ N} \quad \Rightarrow \quad T_s = \frac{(8 \text{ kg})(9.8 \text{ m/s}^2)}{\sqrt{2}/2} = 110.87 \text{ N}$$

Use these values for λ, μ, and T_s to find f.

$$f = \frac{v}{\lambda} = \frac{\sqrt{T_s/\mu}}{\lambda} = \frac{\sqrt{(110.87 \text{ N})/(0.02652 \text{ kg/m})}}{5.657 \text{ m}} = 11 \text{ Hz}$$

Assess: This seems like a reasonable frequency for a mechanical system like this.

21.46. Model: Assume that while the spring provides the same tension to both strings it also acts as a fixed point for the end of each string so an integral number of half wavelengths fit in each string.

Visualize: Use a subscript L for the left string and R for the right string. We are given $\mu_L = 2.0$ g/m. From the assumption above we know $(T_s)_L = (T_s)_R = T_s$. We are also given $f_L = f_R = f$ and $L_L = L_R = L$. Notice from the diagram that $\lambda_L = L$ and $\lambda_R = \frac{2}{3}L$.

Solve: From $v = \lambda f$ and $v = \sqrt{T_s/\mu}$ eliminate v and solve for μ: $\mu = T/(\lambda f)^2$.

$$\frac{\mu_R}{\mu_L} = \frac{(T_s)_R/(\lambda_R f_R)^2}{(T_s)_L/(\lambda_L f_L)^2} = \frac{T_s/(\frac{2}{3}Lf)^2}{T_s/(Lf)^2} = \frac{1}{(\frac{2}{3})^2} = \frac{9}{4}$$

$$\mu_R = \frac{9}{4}\mu_L = \frac{9}{4}(2.0\text{ g/m}) = 4.5\text{ g/m}$$

Assess: We expect a slower wave speed in the right string to correspond to a larger mass density.

21.47. Model: The microwave forms a standing wave between the two reflectors.

Solve: (a) There are reflectors at both ends, so the electromagnetic standing wave acts just like the standing wave on a string that is tied at both ends. The frequencies of the standing waves are

$$f_m = m\frac{v_{\text{light}}}{2L} = m\frac{c}{2L} = m\frac{3.0\times10^8 \text{ m/s}}{2(0.10 \text{ m})} = m(1.5\times10^9 \text{ Hz}) = 1.5m \text{ GHz}$$

where we have noted that electromagnetic waves of all frequencies travel with the speed of light c. The generator can produce standing waves at any frequency between 10 GHz and 20 GHz. These are

m	f_m (GHz)
7	10.5
8	12.0
9	13.5
10	15.0
11	16.5
12	18.0
13	19.5

(b) There are 7 different standing wave frequencies. Even-numbered values of n create a node at the center, and odd-numbered values of n create an antinode at the center. So the frequencies where the midpoint is an antinode are 10.5, 13.5, 16.5, and 19.5 GHz.

21.48. Model: The fundamental wavelength of an open-open tube is $2L$ and that of an open-closed tube is $4L$.

Solve: We are given that

$$f_{1\text{ open-closed}} = f_{3\text{ open-open}} = 3f_{1\text{ open-open}}$$

$$\Rightarrow \frac{v_{\text{air}}}{\lambda_{1\text{ open-closed}}} = 3\frac{v_{\text{air}}}{\lambda_{1\text{ open-open}}} \Rightarrow \frac{1}{4L_{\text{open-closed}}} = \frac{3}{2L_{\text{open-open}}}$$

$$\Rightarrow L_{\text{open-closed}} = \frac{2L_{\text{open-open}}}{12} = \frac{2(78.0\text{ cm})}{12} = 13.0\text{ cm}$$

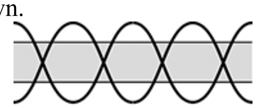
21.49. Model: A tube forms standing waves.

Solve: (a) The fundamental frequency cannot be 390 Hz because 520 Hz and 650 Hz are not integer multiples of it. But we note that the *difference* between 390 Hz and 520 Hz is 130 Hz as is the *difference* between 520 Hz and 650 Hz. We see that 390 Hz = 3 × 130 Hz = $3f_1$, 520 Hz = $4f_1$, and 650 Hz = $5f_1$. So we are seeing the third, fourth, and fifth harmonics of a tube whose fundamental frequency is 130 Hz. According to Equation 21.17, this is an open-open tube because $f_m = mf_1$ with m = 1, 2, 3, 4, … For an open-closed tube m has only odd values.

(b) Knowing f_1, we can now find the length of the tube:

$$L = \frac{v}{2f_1} = \frac{343 \text{ m/s}}{2(130 \text{ Hz})} = 1.32 \text{ m}$$

(c) 520 Hz is the fourth harmonic. This is a sound wave, not a wave on a string, so the wave will have four nodes and will have antinodes at the ends, as shown.



520 Hz $m = 4$

(d) With carbon dioxide, the new fundamental frequency is

$$f_1 = \frac{v}{2L} = \frac{280 \text{ m/s}}{2(1.32 \text{ m})} = 106 \text{ Hz}$$

Thus the frequencies of the n = 3, 4, and 5 modes are $f_3 = 3f_1 = 318$ Hz, $f_4 = 4f_1 = 424$ Hz, and $f_5 = 5f_1 = 530$ Hz.

21.50. Model: Particles of the medium at the nodes of a standing wave have zero displacement.
Solve: The cork dust settles at the nodes of the sound wave where there is no motion of the air molecules. The separation between the centers of two adjacent piles is $\frac{1}{2}\lambda$. Thus,

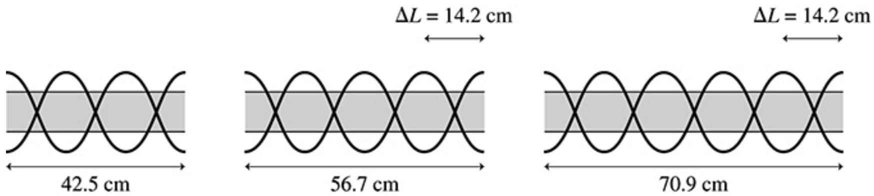
$$\frac{123 \text{ cm}}{3} = \frac{\lambda}{2} \Rightarrow \lambda = 82 \text{ cm}$$

Because the piston is driven at a frequency of 400 Hz, the speed of the sound wave in oxygen is

$$v = f\lambda = (400 \text{ Hz})(0.82 \text{ m}) = 328 \text{ m/s}$$

Assess: A speed of 328 m/s in oxygen is close to the speed of sound in air, which is 343 m/s at 20°C.

21.51. **Model:** The nodes of a standing wave are spaced $\lambda/2$ apart.
Visualize:



Solve: The wavelength of the mth mode of an open-open tube is $\lambda_m = 2L/m$. Or, equivalently, the length of the tube that generates the mth mode is $L = m(\lambda/2)$. Here λ is the same for all modes because the frequency of the tuning fork is unchanged. Increasing the length of the tube to go from mode m to mode $m + 1$ requires a length change

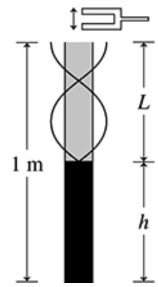
$$\Delta L = (m+1)(\lambda/2) - m\lambda/2 = \lambda/2$$

That is, lengthening the tube by $\lambda/2$ adds an additional antinode and creates the next standing wave. This is consistent with the idea that the nodes of a standing wave are spaced $\lambda/2$ apart. This tube is first increased $\Delta L = 56.7 \text{ cm} - 42.5 \text{ cm} = 14.2 \text{ cm}$, then by $\Delta L = 70.9 \text{ cm} - 56.7 \text{ cm} = 14.2 \text{ cm}$. Thus $\lambda/2 = 14.2$ cm and thus $\lambda = 28.4 \text{ cm} = 0.284$ m. Therefore the frequency of the tuning fork is

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.284 \text{ m}} = 1208 \text{ Hz} \approx 1210 \text{ Hz}$$

21.52. **Model:** The open-closed tube forms standing waves.

Visualize:



Solve: When the air column length L is the proper length for a 580 Hz standing wave, a standing wave resonance will be created and the sound will be loud. From Equation 21.18, the standing wave frequencies of an open-closed tube are $f_m = m(v/4L)$, where v is the speed of sound in air and m is an *odd* integer: m = 1, 3, 5, … The frequency is fixed at 580 Hz, but as the length L changes, 580 Hz standing waves will occur for different values of m. The length that causes the mth standing wave mode to be at 580 Hz is

$$L = \frac{m(343 \text{ m/s})}{(4)(580 \text{ Hz})}$$

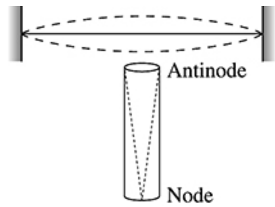
We can place the values of L, and corresponding values of $h = 1 \text{ m} - L$, in a table:

m	L	$h = 1 \text{ m} - L$
1	0.148 m	0.852 m = 85.2 cm
3	0.444 m	0.556 m = 55.6 cm
5	0.739 m	0.261 m = 26.1 cm
7	1.035 m	h can't be negative

So water heights of 26 cm, 56 cm, and 85 cm will cause a standing wave resonance at 580 Hz. The figure shows the $m = 3$ standing wave at $h = 56$ cm.

21.53. Model: A stretched wire, which is fixed at both ends, forms a standing wave whose fundamental frequency $f_{1\text{ wire}}$ is the same as the fundamental frequency $f_{1\text{ open-closed}}$ of the open-closed tube. The two frequencies are the same because the oscillations in the wire drive oscillations of the air in the tube.

Visualize:



Solve: The fundamental frequency in the wire is

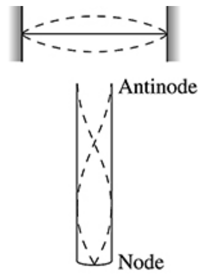
$$f_{1\text{ wire}} = \frac{v_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{T_{\text{S}}}{\mu}} = \frac{1}{(1.0\text{ m})}\sqrt{\frac{440\text{ N}}{(0.0010\text{ kg}/0.050\text{ m})}} = 469\text{ Hz}$$

The fundamental frequency in the open-closed tube is

$$f_{1\text{ open-closed}} = 469\text{ Hz} = \frac{v_{\text{air}}}{4L_{\text{tube}}} = \frac{340\text{ m/s}}{4L_{\text{tube}}} \Rightarrow L_{\text{tube}} = \frac{340\text{ m/s}}{4(469\text{ Hz})} = 0.181\text{ m} \approx 18\text{ cm}$$

21.54. Model: A stretched wire, which is fixed at both ends, creates a standing wave whose fundamental frequency is $f_{1\text{ wire}}$. The second vibrational mode of an open-closed tube is $f_{3\text{ open-closed}}$. These two frequencies are equal because the wire's vibrations generate the sound wave in the open-closed tube.

Visualize:



Solve: The frequency in the tube is

$$f_{3\text{ open-closed}} = \frac{3v_{\text{air}}}{4L_{\text{tube}}} = \frac{3(340\text{ m/s})}{4(0.85\text{ cm})} = 300\text{ Hz}$$

$$\Rightarrow f_{1\text{ wire}} = 300\text{ Hz} = \frac{v_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{T_{\text{S}}}{\mu}}$$

$$\Rightarrow T_{\text{S}} = (300\text{ Hz})^2(2L_{\text{wire}})^2\mu = (300\text{ Hz})^2\,(2 \times 0.25\text{ m})^2(0.020\text{ kg/m}) = 450\text{ N}$$

21.55. Model: The standing waves in the tube will have a displacement antinode at the top where the gas molecules are free to move and a node at the water where they are not.

Visualize: For the wire, we are given $T_s = 400$ N. In the fundamental mode with both ends of the wire fixed $L_{wire} = \lambda_{wire}/2$. Hence, $\lambda_{wire} = 2L_{wire} = 2(50.0 \text{ cm}) = 1.00 \text{ m}$. We also know $\mu_{wire} = 0.00100 \text{ kg}/0.500 \text{ m} = 0.00200 \text{ kg/m}$. The given information is sufficient to compute the frequency. From $v = \lambda f$ and $v = \sqrt{T_s/\mu}$ eliminate v and solve for f.

$$f = \frac{\sqrt{(T_s)_{wire}/\mu_{wire}}}{\lambda_{wire}} = \frac{\sqrt{400 \text{ N}/(0.00200 \text{ kg/m})}}{1.00 \text{ m}} = 447.2 \text{ Hz}$$

Solve: Now we turn our attention to the gas, realizing that the frequency of the wave in the wire will be the same as the frequency of the sound in the gas. There is initially a node of the standing sound wave at the water level in the tube. The water is then lowered until the next standing wave is achieved; this is the next time there is a node at the water level. The distance between adjacent nodes in a standing wave is $\frac{1}{2}\lambda$, so $\Delta h = \frac{1}{2}\lambda_{gas} = 30.5 \text{ cm} \Rightarrow \lambda_{gas} = 61.0 \text{ cm}$.

$$v_{gas} = \lambda_{gas} f_{gas} = (0.610 \text{ m})(447.2 \text{ Hz}) = 273 \text{ m/s}$$

Assess: We were told the gas is more dense than air so it will stay in the tube; for more dense gases we expect a slower sound speed. Our answer bears this out, but is still in the range of the speed of sound for typical gases.

21.56. Model: In a rod in which a longitudinal standing wave can be created, the standing wave is equivalent to a sound standing wave in an open-open tube. Both ends of the rod are antinodes, and the rod is vibrating in the fundamental mode.

Solve: Since the rod is in the fundamental mode, $\lambda_1 = 2L = 2(2.0\text{ m}) = 4.0\text{ m}$. Using the speed of sound in aluminum, the frequency is

$$f_1 = \frac{v_{\text{Al}}}{\lambda_1} = \frac{6420\text{ m/s}}{4.0\text{ m}} = 1605\text{ Hz} \approx 1600\text{ Hz}$$

21.57. Model: Model the tunnel as an open-closed tube.

Visualize: We are given $v = 335$ m/s. We would like to use $f_m = m\dfrac{v}{4L}(m = \text{odd})$ to find L, but we need to know m first. Since m takes on only odd values for the open-closed tube the next resonance after m is $m+2$. We are given $f_m = 4.5\text{ Hz}$ and $f_{m+2} = 6.3\text{ Hz}$.

Solve:

$$\frac{f_{m+2}}{f_m} = \frac{(m+2)\dfrac{v}{4L}}{(m)\dfrac{v}{4L}} = \frac{m+2}{m}$$

$$m\left(\frac{f_{m+2}}{f_m}\right) = m+2$$

$$m\left(\frac{f_{m+2}}{f_m} - 1\right) = 2$$

$$m = \frac{2}{\dfrac{f_{m+2}}{f_m} - 1} = \frac{2}{\dfrac{6.3\text{ Hz}}{4.5\text{ Hz}} - 1} = 5$$

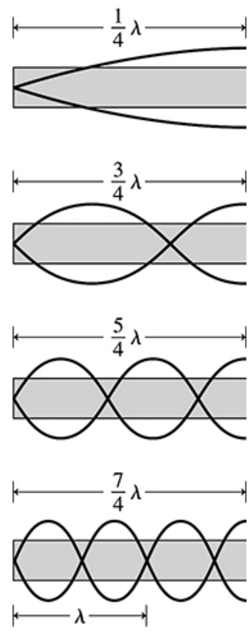
Now that we know m we can finish up.

$$f_m = m\frac{v}{4L} \Rightarrow L = m\frac{v}{4f_m} = (5)\frac{335\text{ m/s}}{4(4.5\text{ Hz})} = 93\text{ m}$$

Assess: 93 m seems like a reasonable length for a tunnel.

21.58. Model: A standing wave in an open-closed tube must have a node at the closed end of the tube and an antinode at the open end.

Visualize:



Solve: We first draw a series of pictures showing all the possible standing waves. By examination, we see that the first standing wave mode is $\frac{1}{4}$ of a wavelength, so the tube's length is $L = \frac{1}{4}\lambda$. The next mode is $\frac{3}{4}$ of a wavelength. The tube's length hasn't changed, so in this mode $L = \frac{3}{4}\lambda$. The next mode is now slightly more than a wavelength. That is, $L = \frac{5}{4}\lambda$. The next mode is $\frac{7}{4}$ of a wavelength, so $L = \frac{7}{4}\lambda$. We see that there is a pattern. The length of the tube and the possible standing wave wavelengths are related by

$$L = \frac{m\lambda}{4} \quad m = 1, 3, 5, 7, \ldots = \text{odd integers}$$

Solving for λ, we find that the wavelengths and frequencies of standing waves in an open-closed tube are

$$\left.\begin{aligned} \lambda_m &= \frac{4L}{m} \\ f_m &= \frac{v}{\lambda_m} = m\frac{v}{4L} \end{aligned}\right\} m = 1,\ 3,\ 5,\ 7,\ \ldots = \text{ odd integers}$$

21.59. Model: The amplitude is determined by the interference of the two waves.

Solve: For interference in one dimension, where the speakers are separated by a distance Δx, the amplitude of the net wave is $A = 2a\cos\left(\frac{1}{2}\Delta\phi\right)$, where a is the amplitude of each wave and $\Delta\phi = 2\pi\Delta x/\lambda + \Delta\phi_0$ is the phase difference between the two waves. The speakers are emitting identical waves so they have identical phase constants and $\Delta\phi_0 = 0$. Thus,

$$A = 1.5a = 2a\cos\left(\frac{\pi\Delta x}{\lambda}\right) \Rightarrow \Delta x = \frac{\lambda}{\pi}\cos^{-1}\left(\frac{1.5}{2}\right)$$

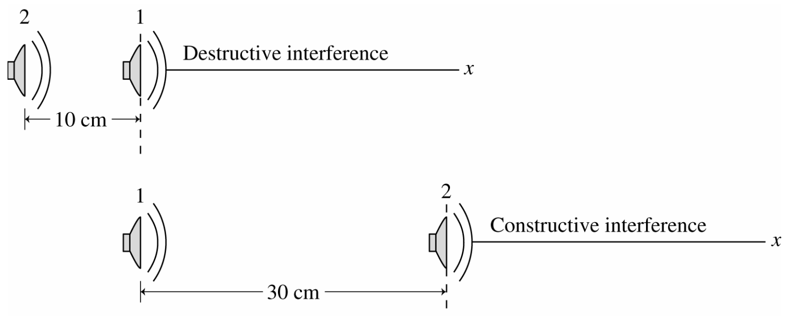
The wavelength of a 1000 Hz tone is $\lambda = v_{\text{sound}}/f = 0.343\text{ m}$. Thus the separation must be

$$\Delta x = \frac{0.343\text{ m}}{\pi}\cos^{-1}(0.75) = 0.0789\text{ m} \approx 7.9\text{ cm}$$

It is essential to note that the argument of the arccosine is in radians, *not* in degrees.

21.60. Model: Constructive or destructive interference occurs according to the phases of the two waves.

Visualize:



Solve: **(a)** To go from destructive to constructive interference requires moving the speaker $\Delta x = \frac{1}{2}\lambda$, equivalent to a phase change of π rad. Since $\Delta x = 40$ cm, we find $\lambda = 80$ cm.

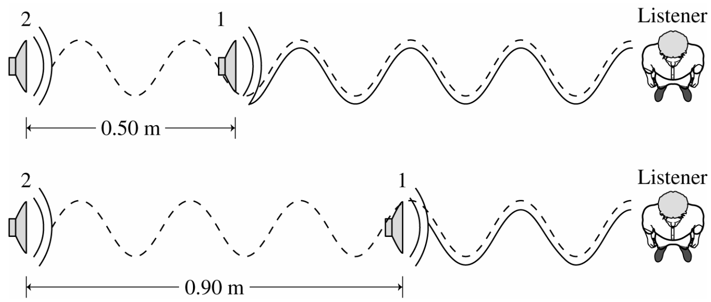
(b) Destructive interference at $\Delta x = 10$ cm requires

$$2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\pi\left(\frac{10\text{ cm}}{80\text{ cm}}\right)\text{ rad} + \Delta\phi_0 = \pi\text{ rad} \Rightarrow \Delta\phi_0 = \frac{3\pi}{4}\text{ rad}$$

(c) When side by side, with $\Delta x = 0$, the phase difference is $\Delta\phi = \Delta\phi_0 = 3\pi/4$ rad. The amplitude of the superposition of the two waves is

$$a = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right| = \left|2a\cos\frac{3\pi}{8}\right| = 0.77a$$

21.61. Model: Interference occurs according to the difference between the phases of the two waves.
Visualize:



Solve: **(a)** The phase difference between the sound waves from the two speakers is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

We have a maximum intensity when $\Delta x = 0.50$ m and $\Delta x = 0.90$ m. This means

$$2\pi\frac{(0.50\text{ m})}{\lambda} + \Delta\phi_0 = 2m\pi \text{ rad} \qquad 2\pi\left(\frac{0.90\text{ m}}{\lambda}\right) + \Delta\phi_0 = 2(m+1)\pi \text{ rad}$$

Taking the difference of the above two equations,

$$2\pi\left(\frac{0.40\text{ m}}{\lambda}\right) = 2\pi \Rightarrow \lambda = 0.40\text{ m} \Rightarrow f = \frac{v_{\text{sound}}}{\lambda} = \frac{340\text{ m/s}}{0.40\text{ m}} = 850\text{ Hz}$$

(b) Using again the equations that correspond to constructive interference,

$$2\pi\left(\frac{0.50\text{ m}}{0.40\text{ m}}\right) + \Delta\phi_0 = 2m\pi \text{ rad} \Rightarrow \Delta\phi_0 = \phi_{20} - \phi_{10} = -\frac{\pi}{2} \text{ rad}$$

We have taken $m = 1$ in the last equation. This is because we always specify phase constants in the range $-\pi$ rad to π rad (or 0 rad to 2π rad). $m = 1$ gives $-\frac{1}{2}\pi$ rad (or equivalently, $m = 2$ will give $\frac{3}{2}\pi$ rad).

21.62. Model: Constructive or destructive interference occurs according to the phases of the two waves.

Solve: The phase difference between the sound waves from the two speakers is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

With no delay between the two signals, $\Delta\phi_0 = 0$ rad and

$$\Delta\phi = \frac{2\pi(2.0 \text{ m})}{v/f} = 2\pi(2.0 \text{ m})\left(\frac{340 \text{ Hz}}{340 \text{ m/s}}\right) = 4\pi \text{ rad}$$

According to Equation 21.22, this corresponds to constructive interference. A delay of 1.47 ms corresponds to an inherent phase difference of

$$\Delta\phi_0 = \left(\frac{2\pi}{T}\right)(1.47 \text{ ms}) \text{ rad } = (2\pi f)(1.47 \text{ ms}) \text{ rad} = 2\pi(1.47 \text{ ms})(340 \text{ Hz}) \text{ rad} = \pi \text{ rad}$$

The phase difference $\Delta\phi$ between the signals is then

$$\Delta\phi = 2\pi\left(\frac{\Delta x}{\lambda}\right) + \Delta\phi_0 = 4\pi \text{ rad} + \pi \text{ rad} = 5\pi \text{ rad}$$

Thus, the interference along the *x*-axis will be perfect destructive.

21.63. Model: Reflection is maximized for constructive interference of the two reflected waves, but minimized for destructive interference.

Solve: **(a)** Constructive interference of the reflected waves occurs for wavelengths given by Equation 21.32:

$$\lambda_m = \frac{2nd}{m} = \frac{2(1.42)(500 \text{ nm})}{m} = \frac{(1420 \text{ nm})}{m}$$

Thus, $\lambda_1 = 1420$ nm, $\lambda_2 = \frac{1}{2}(1420 \text{ nm}) = 710$ nm, $\lambda_3 = 473$ nm, $\lambda_4 = 355$ nm, … Only the wavelength of 473 nm is in the visible range.

(b) For destructive interference of the reflected waves,

$$\lambda = \frac{2nd}{m-\frac{1}{2}} = \frac{2(1.42)(500 \text{ nm})}{m-\frac{1}{2}} = \frac{1420 \text{ nm}}{m-\frac{1}{2}}$$

Thus, $\lambda_1 = 2 \times 1420 \text{ nm} = 2840$ nm, $\lambda_2 = \frac{2}{3}(1420 \text{ nm}) = 947$ nm, $\lambda_3 = 568$ nm, $\lambda_4 = 406$ nm, … The wavelengths of 406 nm and 568 nm are in the visible range.

(c) Beyond the limits 430 nm and 690 nm the eye's sensitivity drops to about 1 percent of its maximum value. The reflected light is enhanced in blue (473 nm). The transmitted light at mostly 568 nm will be yellowish green.

21.64. Solve: (a) The intensity of reflected light from the uncoated glass is $I_0 = ca^2$, where a is the amplitude of the reflected light. We will assume that the amplitude of the reflected light from both the bottom and the top of the coated film is a. The interference of the two reflected waves determines the amplitude of the resultant wave which is given by

$$A = \left|2a\cos(\Delta\phi/2)\right| \quad \text{where} \qquad \Delta\phi = 2\pi\frac{\Delta x}{\lambda_{\text{coat}}} + \Delta\phi_0$$

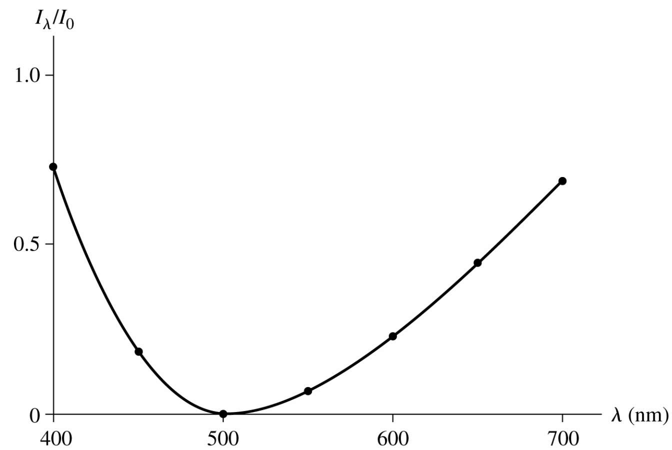
With $\Delta\phi_0 = 0$ rad, $\Delta x = 2d$, and $\lambda_{\text{coat}} = \lambda_{\text{air}}/n$, we have

$$\Delta\phi = \frac{2\pi(2d)}{\lambda_{\text{air}}/n} + 0\text{ rad} = \frac{4\pi dn}{\lambda} = \frac{4\pi(92\text{ nm})(1.39)}{\lambda} = \frac{1607\text{ nm}}{\lambda}$$

$$\Rightarrow A = \left|2a\cos\frac{803.5\text{ nm}}{\lambda}\right| \Rightarrow I_\lambda = cA^2 = c4a^2\cos^2\left(\frac{803.5\text{ nm}}{\lambda}\right) \Rightarrow \frac{I_\lambda}{I_0} = 4\cos^2\left(\frac{803.5\text{ nm}}{\lambda}\right)$$

(b) The values of (I_λ/I_0) at $\lambda = 400$, 450, 500, 550, 600, 650, and 700 nm are 0.719, 0.182, 0.005, 0.048, 0.211, 0.431, and 0.674, respectively.

(c)



21.65. Model: Reflection is minimized when the two reflected waves interfere destructively.

Solve: Equation 21.2 gives the condition for perfect destructive interference between the two waves:

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\left(m + \tfrac{1}{2}\right)\pi \text{ rad}$$

The wavelength of the sound is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1200 \text{ Hz}} = 0.2858 \text{ m}$$

Let d be the separation between the mesh and the wall. Substituting $\Delta\phi_0 = 0$ rad, $\Delta x = 2d$, $m = 0$, and the above value for the wavelength,

$$\frac{2\pi(2d)}{0.2858 \text{ m}} + 0 \text{ rad} = \pi \text{ rad} \Rightarrow d = \frac{0.2858 \text{ m}}{4} = 0.0715 \text{ m} = 7.15 \text{ cm}$$

21.66. Model: A light wave that reflects from a boundary at which the index of refraction increases has a phase shift of π rad.

Solve: **(a)** Because $n_{\text{film}} > n_{\text{air}}$, the wave reflected from the outer surface of the film (called 1) is inverted due to the phase shift of π rad. The second reflected wave does not go through any phase shift of π rad because the index of refraction decreases at the boundary where this wave is reflected, which is on the inside of the soap film. We can write for the phases

$$\phi_1 = kx_1 + \phi_{10} + \pi \text{ rad} \qquad \phi_2 = kx_2 + \phi_{20} + 0 \text{ rad}$$

$$\Rightarrow \Delta\phi = \phi_2 - \phi_1 = k(x_2 - x_1) + (\phi_{20} - \phi_{10}) - \pi \text{ rad} = k\Delta x + \Delta\phi_0 - \pi \text{ rad} = k\Delta x - \pi \text{ rad}$$

$\Delta\phi_0 = 0$ rad because the sources are identical. For constructive interference,

$$\Delta\phi = 2m\pi \text{ rad} \Rightarrow k\Delta x - \pi \text{ rad} = 2m\pi \text{ rad} \Rightarrow \left(\frac{2\pi}{\lambda_{\text{film}}}\right)(2d) = (2m+1)\pi \text{ rad}$$

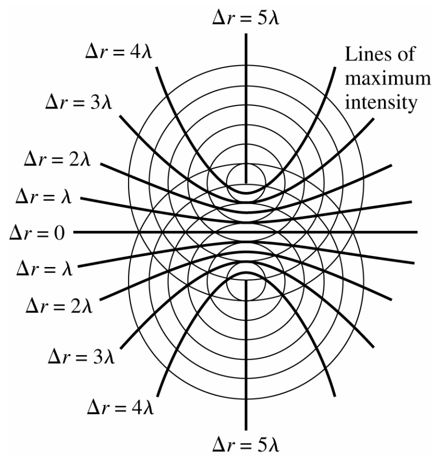
$$\Rightarrow \lambda_{\text{film}} = \frac{\lambda_C}{n} = \frac{2d}{m+\frac{1}{2}} \Rightarrow \lambda_C = \frac{2nd}{m+\frac{1}{2}} = \frac{2.66d}{m+\frac{1}{2}} \quad m = 0, 1, 2, 3, \ldots$$

(b) For $m = 0$ the wavelength for constructive interference is

$$\lambda_C = \frac{(2.66)(390 \text{ nm})}{\left(\frac{1}{2}\right)} = 2075 \text{ nm}$$

For $m = 1$ and 2, $\lambda_C = 692 \text{ nm}$ (~red) and $\lambda_C = 415 \text{ nm}$ (~violet). Red and violet together give a purplish color.

21.67. Model: The two radio antennas are sources of in-phase, circular waves. The overlap of these waves causes interference.
Visualize:



Solve: Maxima occur along lines such that the path difference to the two antennas is $\Delta r = m\lambda$. The 750 MHz = 7.50×10^8 Hz wave has a wavelength $\lambda = c/f = 0.40$ m. Thus, the antenna spacing $d = 2.0$ m is exactly 5λ. The maximum possible intensity is on the line connecting the antennas, where $\Delta r = d = 5\lambda$. So this is a line of maximum intensity. Similarly, the line that bisects the two antennas is the $\Delta r = 0$ line of maximum intensity. In between, in each of the four quadrants, are four lines of maximum intensity with $\Delta r = \lambda$, 2λ, 3λ, and 4λ. Although we have drawn a fairly accurate picture, you do *not* need to know precisely where these lines are located to know that you *have* to cross them if you walk all the way around the antennas. Thus, you will cross 20 lines where $\Delta r = m\lambda$ and will detect 20 maxima.

21.68. Model: The changing sound intensity is due to the interference of two overlapped sound waves.

Solve: Minimum intensity implies destructive interference. Destructive interference occurs where the path length difference for the two waves is $\Delta r = \left(m+\frac{1}{2}\right)\lambda$. We have assumed $\Delta\phi_0 = 0$ rad for two speakers playing "exactly the same" tone. The wavelength of the sound is $\lambda = v_{\text{sound}}/f = (343 \text{ m/s})/686 \text{ Hz} = 0.500 \text{ m}$. Consider a point that is a distance x in front of the top speaker. Let r_1 be the distance from the top speaker to the point and r_2 the distance from the bottom speaker to the point. We have

$$r_1 = x \qquad\qquad r_2 = \sqrt{x^2 + (3 \text{ m})^2}$$

Destructive interference occurs at distances x such that

$$\Delta r = \sqrt{x^2 + 9 \text{ m}^2} - x = \left(m+\tfrac{1}{2}\right)\lambda$$

To solve for x, isolate the square root on one side of the equation and then square:

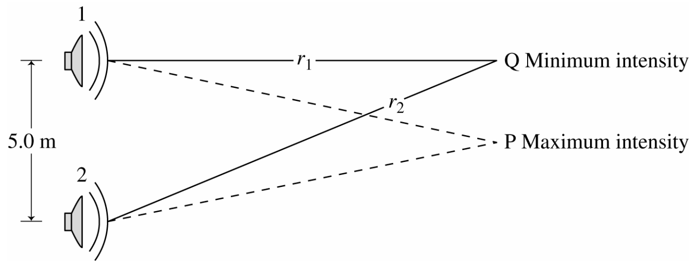
$$x^2 + 9 \text{ m} = \left[x + \left(m+\tfrac{1}{2}\right)\lambda\right]^2 = x^2 + 2\left(m+\tfrac{1}{2}\right)\lambda x + \left(m+\tfrac{1}{2}\right)^2\lambda^2 \Rightarrow x = \frac{9 \text{ m} - \left(m+\frac{1}{2}\right)^2\lambda^2}{2\left(m+\frac{1}{2}\right)\lambda}$$

Evaluating x for different values of m:

m	x (m)
0	17.88
1	5.62
2	2.98
3	1.79

Because you start at $x = 2.5$ m and walk *away* from the speakers, you will only hear minima for values $x > 2.5$ m. Thus, to correct significant figures, minima will occur at distances of 3.0 m, 5.6 m, and 18 m.

21.69. Model: The changing sound intensity is due to the interference of two overlapped sound waves.
Visualize: The listener moving relative to the speakers changes the phase difference between the waves.



Solve: **(a)** Initially when you are at *P*, equidistant from the speakers, you hear a sound of maximum intensity. This implies that the two speakers are in phase ($\Delta\phi_0 = 0$). However, on moving to *Q* you hear a minimum of sound intensity implying that the path length difference from the two speakers to *Q* is $\lambda/2$. Thus,

$$\tfrac{1}{2}\lambda = \Delta r = \sqrt{(r_1)^2 + (5.0\text{ m})^2} - r_1 = \sqrt{(12.0\text{ m})^2 + (5.0\text{ m})^2} - 12.0\text{ m} = 1.0\text{ m}$$

$$\Rightarrow \lambda = 2.0\text{ m} \;\Rightarrow f = \frac{v}{\lambda} = \frac{340\text{ m/s}}{2.0\text{ m}} = 170\text{ Hz}$$

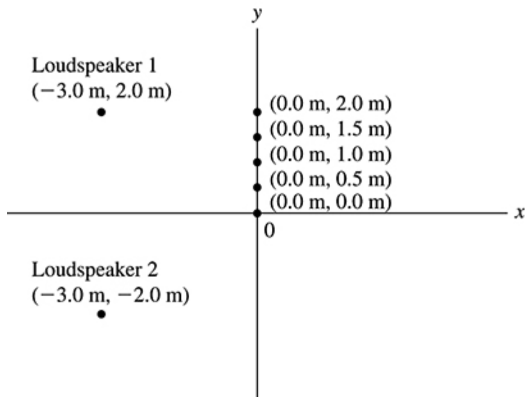
(b) At *Q*, the condition for perfect destructive interference is

$$\Delta\phi = \frac{2\pi(\Delta r)}{\lambda} + 0\text{ rad} = 2\left(m - \tfrac{1}{2}\right)\pi\text{ rad} \Rightarrow \frac{2\pi\Delta r}{v/f} = 2\left(m - \tfrac{1}{2}\right)\pi\text{ rad}$$

$$\Rightarrow f = \left(m - \tfrac{1}{2}\right)\frac{v}{\Delta r} = \left(m - \tfrac{1}{2}\right)\left(\frac{340\text{ m/s}}{1.0\text{ m}}\right)$$

For m = 1, 2, and 3, $f_1 = 170$ Hz, $f_2 = 510$ Hz, and $f_3 = 850$ Hz.

21.70. Model: The amplitude is determined by the interference of the two waves.
Visualize:



Solve: The amplitude of the sound wave is $A = \left|2a\cos\left(\tfrac{1}{2}\Delta\phi\right)\right|$. With $\Delta\phi_0 = 0$ rad, the phase difference between the waves is

$$\Delta\phi = \phi_2 - \phi_1 = 2\pi\frac{\Delta r}{\lambda} = 2\pi\frac{\Delta r}{2.0\text{ m}} \Rightarrow A = \left|2a\cos\left(\frac{\pi\Delta r}{2.0\text{ m}}\right)\right|$$

At the coordinates (0.0 m, 0.0 m), $\Delta r = 0$ m, so $A = 2a$. At the coordinates (0.0 m, 0.5 m),

$$\Delta r = \sqrt{(3.0\text{ m})^2 + (2.5\text{ m})^2} - \sqrt{(3.0\text{ m})^2 + (1.5\text{ m})^2} = 0.551\text{ m} \Rightarrow A = \left|2a\cos\frac{(0.551\text{ m})\pi}{2.0\text{ m}}\right| = 1.30a$$

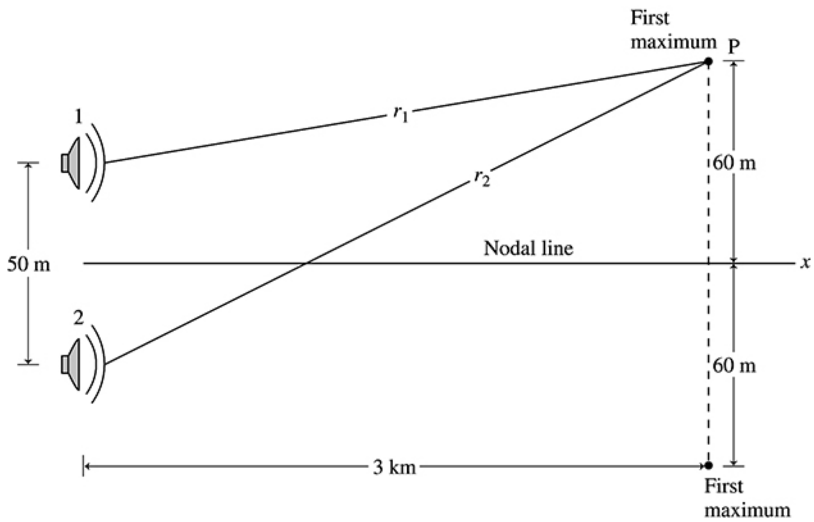
At the coordinates (0.0 m, 1.0 m),

$$\Delta r = \sqrt{(3.0\text{ m})^2 + (3.0\text{ m})^2} - \sqrt{(3.0\text{ m})^2 + (1.0\text{ m})^2} = 1.08\text{ m} \Rightarrow A = \left|2a\cos\left(\frac{(1.08\text{ m})\pi}{2.0\text{ m}}\right)\right| = 0.25a$$

At the coordinates (0.0 m, 1.5 m), $\Delta r = 1.568$ m and $A = 1.56a$. At the coordinates (0.0 m, 2.0 m), $\Delta r =$ 2.0 m and $A = 2a$.

21.71. Model: The two radio transmitters are sources of out-of-phase, circular waves. The overlap of these waves causes interference.

Visualize:



Solve: The phase difference of the waves at point P is given by

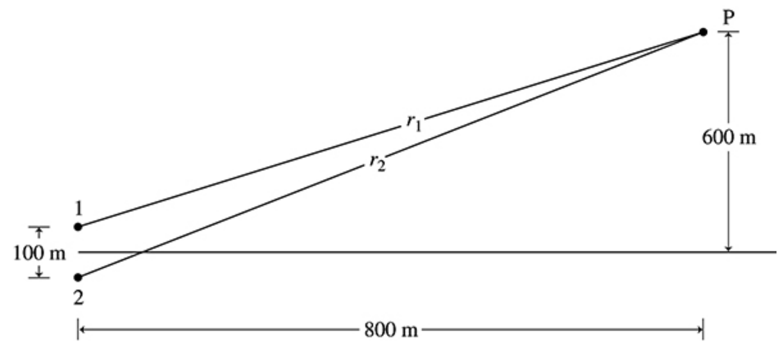
$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0$$

$$\Delta r = \sqrt{(3000 \text{ m})^2 + (85 \text{ m})^2} - \sqrt{(3000 \text{ m})^2 + (35 \text{ m})^2} = 0.99976 \text{ m}$$

The intensity at P is a maximum. Using $m = 1$ for the first maximum, and $\Delta\phi_0 = \pi$ rad since the transmitters are out of phase, the condition for constructive interference is $\Delta\phi = 2m\pi = 2\pi$. Thus,

$$2\pi \text{ rad} = 2\pi\frac{\Delta \text{r}}{\lambda} + \pi \text{ rad} \Rightarrow \lambda = 2\Delta r = 2(0.99976 \text{ m}) \Rightarrow f = \frac{c}{\lambda} = \frac{3\times 10^8 \text{ m/s}}{2(0.99976 \text{ m})} = 150 \text{ MHz}$$

21.72. Model: The two radio antennas are sources of in-phase waves. The overlap of these waves causes interference.
Visualize:



Solve: **(a)** The phase difference of the two waves at point P is given by

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 \qquad \Delta r = \sqrt{(800\text{ m})^2 + (650\text{ m})^2} - \sqrt{(800\text{ m})^2 + (550\text{ m})^2} = 59.96\text{ m}$$

The wavelength of the radio wave is

$$\lambda = \frac{c}{f} = \frac{3.0\times10^8\text{ m/s}}{3.0\times10^6\text{ m}} = 100\text{ m}$$

Since the sources are identical, $\Delta\phi_0 = 0$ rad. The phase difference at P due to the two waves is

$$\Delta\phi = 2\pi\left(\frac{59.96\text{ m}}{100\text{ m}}\right) + 0\text{ rad} = 1.2\pi\text{ rad}$$

(b) Since $\Delta\phi = 1.20\pi = 0.6(2\pi)$, which is neither $m2\pi$ nor $(m+\frac{1}{2})2\pi$, the interference at P is somewhere in between maximum constructive and perfect destructive.
(c) At a point 10 m further north we have

$$\Delta r = \sqrt{(800\text{ m})^2 + (660\text{ m})^2} - \sqrt{(800\text{ m})^2 + (560\text{ m})^2} = 60.58\text{ m}$$

$$\Rightarrow \Delta\phi = 2\pi\left(\frac{60.58\text{ m}}{100\text{ m}}\right) + 0\text{ rad} = 1.21\pi\text{ rad} = (0.605)2\pi$$

Because the phase difference is increasing as you move north, you are moving from a destructive interference condition $\Delta\phi = (m+\frac{1}{2})2\pi$ with $m = 0$ toward a constructive interference condition $\Delta\phi = m(2\pi)$ with $m = 1$. The signal strength will therefore increase.

21.73. Model: The amplitude is determined by the interference of the two waves.

Solve: (a) We have three identical loudspeakers as sources. Δr between speakers 1 and 2 is 1.0 m and $\lambda = 2.0$ m. Thus $\Delta r = \frac{1}{2}\lambda$, which gives perfect destructive interference for in-phase sources. That is, the interference of the waves from loudspeakers 1 and 2 is perfect destructive, leaving only the contribution due to speaker 3. Thus the amplitude is a.

(b) If loudspeaker 2 is moved away by one-half of a wavelength or 1.0 m, then all three waves will reach you in phase. The amplitude of the superposed waves will therefore be maximum and equal to $A = 3a$.

(c) The maximum intensity is $I_{\text{max}} = CA^2 = 9Ca^2$. The ratio of the intensity to the intensity of a single speaker is

$$\frac{I_{\text{max}}}{I_{\text{single speaker}}} = \frac{9Ca^2}{Ca^2} = 9$$

21.74. Model: The superposition of two slightly different frequencies gives rise to beats.
Solve: The third harmonic of note A and the second harmonic of note E are

$$f_{3A} = 3f_{1A} = 3(440\text{ Hz}) = 1320\text{ Hz} \qquad f_{2E} = 2f_{1E} = 2(659\text{ Hz}) = 1318\text{ Hz}$$

$$\Rightarrow f_{3A} - f_{2E} = 1320\text{ Hz} - 1318\text{ Hz} = 2\text{ Hz}$$

(b) The beat frequency between the first harmonics is

$$f_{1E} - f_{1A} = 659\text{ Hz} - 440\text{ Hz} = 219\text{ Hz}$$

The beat frequency between the second harmonics is

$$f_{2E} - f_{2A} = 1318\text{ Hz} - 880\text{ Hz} = 438\text{ Hz}$$

The beat frequency between f_{3A} and f_{2E} is 2 Hz. It therefore emerges that the tuner looks for a beat frequency of 2 Hz.
(c) If the beat frequency is 4 Hz, then the second harmonic frequency of the E string is

$$f_{2E} = 1320\text{ Hz} - 4\text{ Hz} = 1316\text{ Hz} \Rightarrow f_{1E} = \tfrac{1}{2}(1316\text{ Hz}) = 658\text{ Hz}$$

Note that the second harmonic frequency of the E string could also be

$$f_{2E} = 1320\text{ Hz} + 4\text{ Hz} = 1324\text{ Hz} \Rightarrow f_{1E} = 662\text{ Hz}$$

This higher frequency can be ruled out because the tuner started with low tension in the E string and we know that

$$v_{\text{string}} = \lambda f = \sqrt{\frac{T}{\mu}} \Rightarrow f \propto \sqrt{T}$$

21.75. Model: The superposition of two slightly different frequencies creates beats.

Solve: **(a)** The wavelength of the sound initially created by the flutist is

$$\lambda = \frac{342 \text{ m/s}}{440 \text{ Hz}} = 0.77727 \text{ m}$$

When the speed of sound inside her flute has increased due to the warming up of the air, the new frequency of the A note is

$$f' = \frac{346 \text{ m/s}}{0.77727 \text{ m}} = 445 \text{ Hz}$$

Thus the flutist will hear beats at the following frequency:

$$f' - f = 445 \text{ Hz} - 440 \text{ Hz} = 5 \text{ beats/s}$$

Note that the wavelength of the A note is determined by the length of the flute rather than the temperature of air or the increased sound speed.

(b) The initial length of the flute is $L = \frac{1}{2}\lambda = \frac{1}{2}(0.77727 \text{ m}) = 0.3886 \text{ m}$. The new length to eliminate beats needs to be

$$L' = \frac{\lambda'}{2} = \frac{1}{2}\left(\frac{v'}{f}\right) = \frac{1}{2}\left(\frac{346 \text{ m/s}}{440 \text{ Hz}}\right) = 0.3932 \text{ m}$$

Thus, she will have to extend the "tuning joint" of her flute by

$$0.3932 \text{ m} - 0.3886 \text{ m} = 0.0046 \text{ m} = 4.6 \text{ mm}$$

21.76. Solve: (a) Yvette's speed is the width of the room divided by time. This means

$$v_Y = \frac{n\left(\frac{1}{2}\lambda\right)}{t} \Rightarrow \frac{n}{t} = \frac{2v_Y}{\lambda}$$

Note that $\frac{1}{2}\lambda$ is the distance between two consecutive antinodes, and n is the number of such half wavelengths that fill the entire width of the room.

(b) Yvette observes a higher frequency f_+ of the source she is moving toward and a lower frequency f_- of the source she is receding from. If v is the speed of sound and f is the sound wave's frequency, we have

$$f_+ = f\left(1+\frac{v_Y}{v}\right) \qquad f_- = f\left(1-\frac{v_Y}{v}\right)$$

The expression for the beat frequency is

$$f_+ - f_- = f\left(1+\frac{v_Y}{v}\right) - f\left(1-\frac{v_Y}{v}\right) = 2f\frac{v_Y}{v} = 2\frac{v}{\lambda}\frac{v_Y}{v} = \frac{2v_Y}{\lambda}$$

(c) The answers to part (a) and (b) are the same. Even though you and Yvette have different perspectives, you should agree as to how many modulations per second she hears.

21.77. Model: The frequency of the loudspeaker's sound in the back of the pick-up truck is Doppler shifted. As the truck moves away from you, its frequency is decreased.

Solve: Because you hear 8 beats per second as the truck drives away from you, the frequency of the sound from the speaker in the pick-up truck is $f_- = 400 \text{ Hz} - 8 \text{ Hz} = 392 \text{ Hz}$. This frequency is

$$f_- = \frac{f_0}{1 + v_S/v} \Rightarrow 1 + \frac{v_S}{343 \text{ m/s}} = \frac{400 \text{ Hz}}{392 \text{ Hz}} = 1.020408 \Rightarrow v_S = 7.0 \text{ m/s}$$

That is, the velocity of the source v_S and hence the pick-up truck is 7.0 m/s.

21.78. Model: A stretched string under tension supports standing waves.

Solve: **(a)** The wave speed on a stretched string is

$$v_{\text{string}} = \sqrt{\frac{T}{\mu}} = f\lambda \Rightarrow f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

The wavelength λ cannot change if the length of the string does not change. So,

$$\frac{df}{dT} = \frac{1}{\lambda}\frac{1}{\sqrt{\mu}}\tfrac{1}{2}(T)^{-1/2} = \frac{1}{2\lambda}\frac{1}{\sqrt{\mu T}} = \frac{1}{2T}\left(\frac{1}{\lambda}\sqrt{\frac{T}{\mu}}\right) = \frac{1}{2T}f \;\Rightarrow \frac{\Delta f}{f} = \frac{\Delta T}{2T}$$

(b) Since there are 5 beats per second,

$$\Delta f = 5\text{ Hz} = \frac{f}{2}\frac{\Delta T}{T} \Rightarrow \frac{\Delta T}{T} = \frac{10\text{ Hz}}{f} = \frac{10\text{ Hz}}{500\text{ Hz}} = 0.020 = 2.0\%$$

That is, an increase of 2.0% in the tension of one of the strings will cause 5 beats per second.

21.79. Model: The microphone will detect a loud sound only if there is a standing wave resonance in the tube. The sound frequency does not change, but changing the length of the tube can create a standing wave.

Solve: The standing wave condition is

$$f = 280 \text{ Hz} = m\frac{v}{2L} \qquad m = 1,\ 2,\ 3,\ldots$$

where L is the total length of the tube. When the slide is extended a distance s, the tube has two straight sides, each of length s + 80 cm, plus a semicircular bend of length $\frac{1}{2}(2\pi r)$. The radius is $r = \frac{1}{2}(10 \text{ cm}) = 5.0 \text{ cm}$. The tube's total length is

$$L = 2(s + 80 \text{ cm}) + \tfrac{1}{2}(2\pi \times 5.0 \text{ cm}) = 175.7 \text{ cm } + 2s = 1.757 \text{ m} + 2s$$

A standing wave resonance will be created if

$$[L = 1.757 + 2s] = \left[m\frac{v}{2f} = m\frac{343 \text{ m/s}}{2(280 \text{ Hz})} = 0.6125m \right]$$

$$\Rightarrow s = 0.3063m - 0.8785 \text{ meters}$$

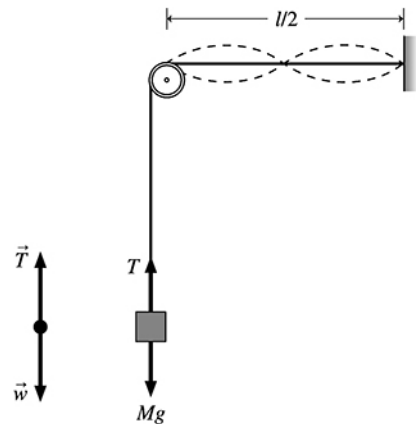
We can tabulate the different extensions s that correspond to standing wave modes m = 1, m = 2, m = 3, and so on.

m	s
1	–0.572 m
2	–0.266 m
3	0.040 m = 4.0 cm
4	0.347 m = 34.7 cm
5	0.653 m = 65.3 cm
6	1.959 m

Physically, the extension must be greater than 0 cm and less than 80 cm. Thus, the three slide extensions that create a standing wave resonance at 280 Hz are 4.0 cm, 35 cm, and 65 cm to two significant figures.

21.80. Model: The stretched wire is vibrating at its second harmonic frequency.

Visualize: Let l be the full length of the wire, and L be the vibrating length of the wire. That is, $L = \left(\frac{1}{2}\right)l$.



Solve: The wave speed on a stretched wire is

$$v_{\text{wire}} = \sqrt{\frac{T_s}{\mu}} = f\lambda$$

The frequency f = 100 Hz and the wavelength $\lambda = \frac{1}{2}l$ because it is a second harmonic wave. The tension T_s = (1.25 kg)g because the hanging mass is in static equilibrium and $\mu = 1.00 \times 10^{-3}$ kg/m. Substituting in these values,

$$\sqrt{\frac{(1.25 \text{ kg})g}{(1.00\times10^{-3} \text{ kg/m})}} = (100 \text{ Hz})\frac{l}{2} \Rightarrow g = (100 \text{ Hz})^2\left(\frac{l}{2}\right)^2\frac{(1.00\times10^{-3} \text{ kg/m})}{(1.25 \text{ kg})} = \left(2.00 \text{ m}^{-1}\text{s}^{-2}\right)l^2$$

To find l we can use the equation for the time period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{T^2}{4\pi^2}g = \frac{(314 \text{ s}/100)^2}{4\pi^2}g = \left(0.250 \text{ s}^2\right)g$$

Substituting this expression for l into the equation for g, we get

$$g = \left(2.000 \text{ m}^{-1}\text{s}^{-2}\right)\left(0.025 \text{ s}^2\right)^2 g^2 \Rightarrow \left(0.125 \text{ m}^{-1}\text{s}^2\right)g^2 - g = 0$$

$$\Rightarrow \left[\left(0.125 \text{ m}^{-1}\text{s}^2\right)g - 1\right]g = 0 \Rightarrow g = 8.00 \text{ m/s}^2$$

Assess: A value of 8.0 m/s^2 is reasonable for the information given in the problem.

21.81. Model: The steel wire is under tension and it vibrates with three antinodes.

Solve: When the spring is stretched 8.0 cm, the standing wave on the wire has three antinodes. This means $\lambda_3 = \frac{2}{3}L$ and the tension T_S in the wire is $T_S = k$ (0.080 m), where k is the spring constant. For this tension,

$$v_{\text{wire}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow f\lambda_3 = \sqrt{\frac{T_S}{\mu}} \Rightarrow f = \frac{3}{2L}\sqrt{\frac{k(0.08\text{ m})}{\mu}}$$

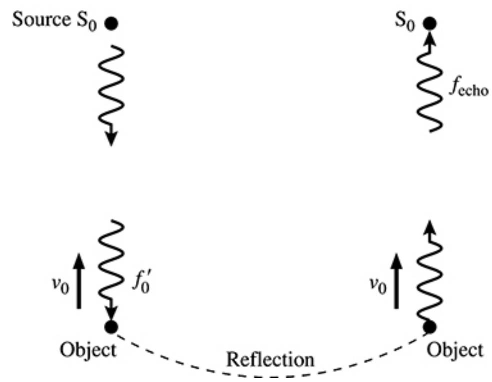
We will let the stretching of the spring be Δx when the standing wave on the wire displays two antinodes. This means $\lambda_2 = L$ and $T_S' = kx$. For the tension T_S',

$$v'_{\text{wire}} = \sqrt{\frac{T_S'}{\mu}} \Rightarrow f\lambda_2 = \sqrt{\frac{T_S'}{\mu}} \Rightarrow f = \frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}}$$

The frequency f is the same in the above two situations because the wire is driven by the same oscillating magnetic field. Now, equating the two frequency equations,

$$\frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}} = \frac{3}{2L}\sqrt{\frac{k(0.080\text{ m})}{\mu}} \Rightarrow \Delta x = 0.18\text{ m} = 18\text{ cm}$$

21.82. Model: The frequency is Doppler shifted to higher values for a detector moving toward the source. The frequency is also shifted to higher values for a source moving toward the detector.
Visualize:



Solve: **(a)** We will derive the formula in two steps. First, the object acts like a moving detector and "observes" a frequency that is given by $f_0' = f_0(1+v_0/v)$. Second, as this moving object reflects (or acts as a "source" of ultrasound waves), the frequency f_{echo} as observed by the original source S_o is $f_{echo} = f_0'(1-v_0/v)^{-1}$. Combining these two equations gives

$$f_{echo} = \frac{f_0'}{1-v_0/v} = \frac{f_0(1+v_0/v)}{1-v_0/v} = \frac{v+v_0}{v-v_0}f_0$$

(b) If $v_0 << v$, then

$$f_{echo} = f_0\left(1+\frac{v_0}{v}\right)\left(1-\frac{v_0}{v}\right)^{-1} = f_0\left(1+\frac{v_0}{v}\right)\left(1+\frac{v_0}{v}+\dots\right) = f_0\left(1+\frac{2v_0}{v}+\dots\right)$$

$$\Rightarrow f_{beat} = f_{echo} - f_0 \approx \frac{2v_0}{v}f_0$$

(c) Using part (b) for the beat frequency,

$$65\text{ Hz} = \left(\frac{2v_0}{1540\text{ m/s}}\right)(2.40\times10^6\text{ Hz}) \Rightarrow v_0 = 2.09\text{ cm/s}$$

(d) Assuming the heart rate is 90 beats per minute the angular frequency is

$$\omega = 2\pi f = 2\pi(1.5\text{ beats/s}) = 9.425\text{ rad/s}$$

Using $v_0 = v_{max} = \omega A$,

$$A = \frac{v_0}{\omega} = \frac{2.09\text{ cm/s}}{9.425\text{ rad/s}} = 2.2\text{ mm}$$

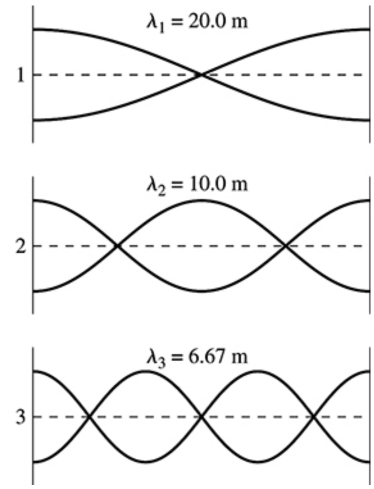
$$T = \frac{(1.50+1.60)}{2(3.0\times10^8\text{ m/s})}\times0.010\text{ m} = 5.17\times10^{-11}\text{ s}$$

21.83. Solve: (a) The wavelengths of the standing wave modes are

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, \ldots$$

$$\Rightarrow \lambda_1 = \frac{2(10.0 \text{ m})}{1} = 20.0 \text{ m} \qquad \lambda_2 = \frac{2(10.0 \text{ m})}{2} = 10.0 \text{ m} \qquad \lambda_3 = \frac{2(10.0 \text{ m})}{3} = 6.67 \text{ m}$$

The depth of the pool is 5.0 m. Clearly the standing waves with λ_2 and λ_3 are "deep water waves" because the 20 m depth is larger than one-quarter of the wavelength. The wave with λ_1 barely qualifies to be a deep water standing wave.



(b) The wave speed for the first standing wave mode is

$$v_1 = \sqrt{\frac{g\lambda_1}{2\pi}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(20.0 \text{ m})}{2\pi}} = 5.59 \text{ m/s}$$

Likewise, $v_2 = 3.95$ m/s and $v_3 = 3.22$ m/s .

(c) We have

$$v = \sqrt{\frac{g\lambda_m}{2\pi}} = f_m\lambda_m \;\Rightarrow f_m = \sqrt{\frac{g}{2\pi\lambda_m}} = \sqrt{\frac{mg}{4\pi L}}$$

Please note that *m* is the mode and not the mass.

(d) The period of oscillation for the first standing wave mode is calculated as follows

$$f_1 = \sqrt{\frac{(1)(9.8 \text{ m/s}^2)}{4\pi(10.0 \text{ m})}} = 0.279 \text{ Hz} \;\Rightarrow\; T_1 = 3.58 \text{ s}$$

Likewise, $T_2 = 2.53$ s and $T_3 = 2.07$ s.

21.84. Model: The overlap of the waves causes interference.

Solve: **(a)** The waves traveling to the left are

$$D_1 = a\sin\left[2\pi\left(-\frac{x}{\lambda}-\frac{t}{T}\right)\right] \qquad D_2 = a\sin\left[2\pi\left(-\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}\right]$$

The phase difference between the waves on the left side of the antenna is thus

$$\Delta\phi_\text{L} = \phi_2-\phi_1 = 2\pi\left(-\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}-2\pi\left(-\frac{x}{\lambda}-\frac{t}{T}\right) = 2\pi\frac{L}{\lambda}+\phi_{20}$$

On the right side of the antennas, where $x_1 = x_2 + L,$ the two waves are

$$D_1 = a\sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right] \qquad D_2 = a\sin\left[2\pi\left(\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}\right]$$

Thus, the phase difference between the waves on the right is

$$\Delta\phi_\text{R} = \phi_2-\phi_1 = 2\pi\left(\frac{(x-L)}{\lambda}-\frac{t}{T}\right)+\phi_{20}-2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right) = -\frac{2\pi L}{\lambda}+\phi_{20}$$

We want to have destructive interference on the country side or on the left and constructive interference on the right. This requires

$$\Delta\phi_\text{L} = \phi_{20}+\frac{2\pi L}{\lambda} = 2\pi\left(m+\tfrac{1}{2}\right) \; \Delta\phi_\text{R} = \phi_{20}-\frac{2\pi L}{\lambda} = 2\pi n$$

These are two simultaneous equations, and we can satisfy them both if L and ϕ_{20} are properly chosen. Subtracting the second equation from the first to eliminate ϕ_{20},

$$\frac{4\pi L}{\lambda} = 2\pi\left(m+\tfrac{1}{2}-n\right) \Rightarrow L = \left(m+\tfrac{1}{2}-n\right)\frac{\lambda}{2}$$

The smallest value of L that works is for $n = m$, in which case $L = \frac{1}{4}\lambda$.

(b) From the $\Delta\phi_\text{R}$ equation,

$$\left[\phi_{20}-\frac{2\pi L}{\lambda} = \phi_{20}-\frac{2\pi\left(\frac{1}{4}\lambda\right)}{\lambda} = \phi_{20}-\frac{\pi}{2}\right] = 2\pi n \Rightarrow \phi_{20} = \frac{\pi}{2}+2\pi n \text{ rad}$$

Adding integer multiples of 2π to the phase constant doesn't really change the wave, so the physically significant phase constant is for $n = 0$, namely $\phi_{20} = \frac{1}{2}\pi$ rad.

(c) We have $\phi_{20} = \frac{1}{2}\pi \text{ rad} = \frac{1}{4}(2\pi)$. If the wave from antenna 2 was delayed by one full period T, it would shift the wave by one full cycle. We would describe this by a phase constant of 2π rad. So a phase constant of $\frac{1}{4}(2\pi)$ rad can be achieved by delaying the wave by $\Delta t = \frac{1}{4}T$.

(d) A wave with frequency $f = 1000 \text{ kHz} = 1.00\times10^{6}$ Hz has a period $T = 1.00\times10^{-6} \text{ s} = 1.00\ \mu\text{s}$ and wavelength $\lambda = c/f = 300$ m. So this broadcast scheme will work if the antennas are spaced L = 75 m apart and if the broadcast from antenna 2 is delayed by $\Delta t = 0.25\ \mu\text{s} = 250$ ns.