Föreläsning 8, mekanik, del 1

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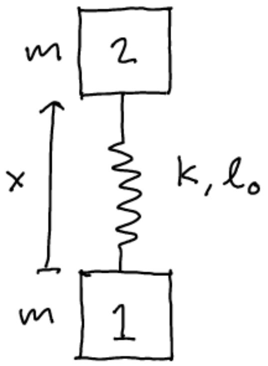


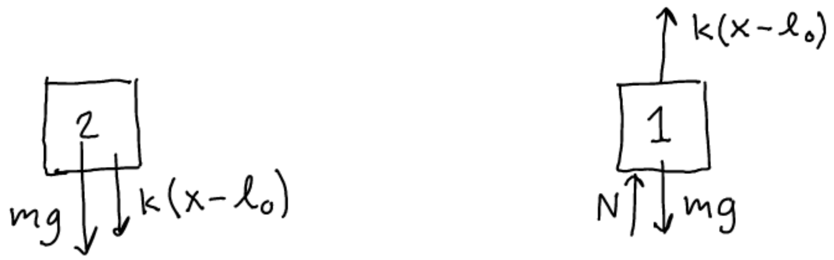
Givet: $k = \frac{4mg}{\ell_0}$

$x(0) = \ell_0/4$

$\dot{x}(0) = 0$

Sökt: t_* då kontakt förloras

Frilägg för $t \le t^*$



Newton II:

$2, \uparrow: -mg - k(x-\ell_0) = m\ddot{x}$

$1, \uparrow: k(x-\ell_0) - mg + N = 0$

jämvikt!

$$(1) \Rightarrow \ddot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = -g + \frac{k\ell_0}{m} \qquad (3)$$

$x = x_h + x_p$.

$x_p = C$

Ins i $(3) \Rightarrow \frac{k}{m} C = -g + \frac{k\ell_0}{m} \Leftrightarrow$

$$\Leftrightarrow \quad C = \frac{-mg}{k} + l_0 = \frac{3l_0}{4}.$$

$$x_h = A\cos\omega_n t + B\sin\omega_n t$$

$$\dot{x}_h = -A\omega_n \sin(\omega_n t) + B\omega_n\cos(\omega_n t)$$

$$BV:\ x(0) = A + \frac{3l_0}{4} = \frac{l_0}{4} \Leftrightarrow A = \frac{-l_0}{2}$$

$$\dot{x}(0) = 0 \Rightarrow B = 0$$

$$\therefore \quad x = \frac{-l_0}{2}\cos(\omega_n t) + \frac{3l_0}{4}$$

$$(2) \Rightarrow N = mg - k(x - l_0) = \ldots = 2mg(1 + \cos(\omega_n t))$$

$$N = 0 \Rightarrow \cos\omega_n t_* = -1 \Rightarrow \omega_n t_* = \pi = t_* = \pi\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{l_0}{g}}$$

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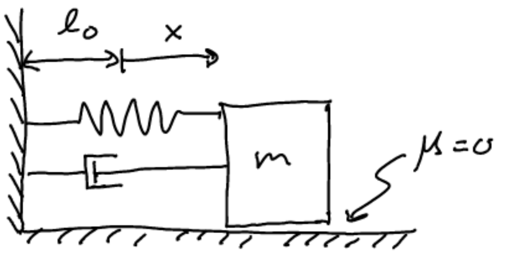


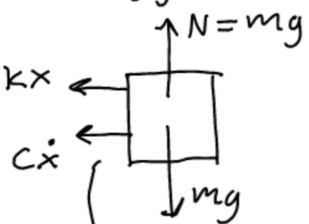
Givet: $x(0) = \frac{l_0}{2}$

$\dot{x}(0) = 0$

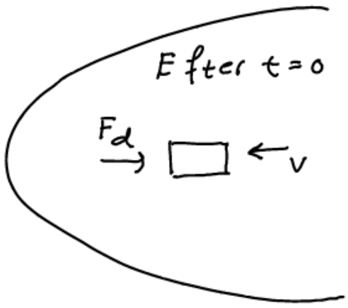
$\delta = 1$

Sökt: l då vänder

Frilägg:



$(\dot{x} < 0 \Rightarrow \rightarrow)$



Newton II

$$\rightarrow: -kx - c\dot{x} = m\ddot{x} \Leftrightarrow \ddot{x} + \underbrace{\frac{c}{m}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2}x = 0$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} < 1$$

$\therefore$ underdämpat system

$$x = x_h = (A\cos(\omega_d \cdot t) + B\sin(\omega_d t))e^{-\zeta\omega_n t}$$

BV: $x(0) = \frac{l_0}{2} \Rightarrow A = \frac{l_0}{2}$

$$\dot{x}(0) = 0 \Rightarrow B\omega_d - \zeta\omega_n A = 0$$

$$\Leftrightarrow B = \frac{\zeta\omega_n l_0}{2\omega_d}$$

$$\therefore x = \left(\frac{l_0}{2}\cos(\omega_d \cdot t) + \frac{\zeta\omega_n l_0}{2\omega_d}\sin(\omega_d \cdot t)\right)e^{-\zeta\omega_n t}$$

I vändläget är $\dot{x} = 0 \Rightarrow$

$$\left(-\frac{l_0}{2}\omega_d\sin(\omega_d \cdot t) + \cancel{\frac{\zeta\omega_n}{2} l_0\cos(\omega_d \cdot t)}\right)e^{-\zeta\omega_n t} -$$

$$-\zeta\omega_n\left(\cancel{\frac{l_0}{2}\cos(\omega_d \cdot t)} + \frac{\zeta\omega_n l_0}{2\omega_d}\cdot\sin(\omega_d \cdot t)\right)e^{-\zeta\omega_n t} = 0$$

$$\Leftrightarrow \sin(\omega_d \cdot t) = 0 \Rightarrow \omega_d t = \pi n$$

$n = 1$ i första vändläget:

$t = \frac{\pi}{\omega_d}$ (Naturligtvis! $t = \frac{T_d}{2}$, T_d period)

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$T_d = \frac{2\pi}{\omega_d}$$

$$x \overset{1}{=} \left(\frac{l_0}{2}\cos(\omega_d t) + \frac{\zeta\omega_n}{2\omega_d}\sin(\omega_d t)\right)\cdot e^{-\zeta\omega_n t}$$

$$\boxed{\omega_d = \sqrt{1-\zeta^2}\,\omega_n}$$

$$(1) \Rightarrow x\left(\frac{\pi}{\omega_d}\right) = \frac{-l_0}{2} e^{-\frac{\zeta\omega_n \pi}{\omega_n\sqrt{1-\zeta^2}}}$$

$$= \frac{-l_0}{2} e^{-\delta/2} = \frac{-l_0}{2} \cdot \frac{1}{\sqrt{e}}$$

$$\therefore \; l = l_0 + x = l_0\left(1 - \frac{1}{2\sqrt{e}}\right)$$

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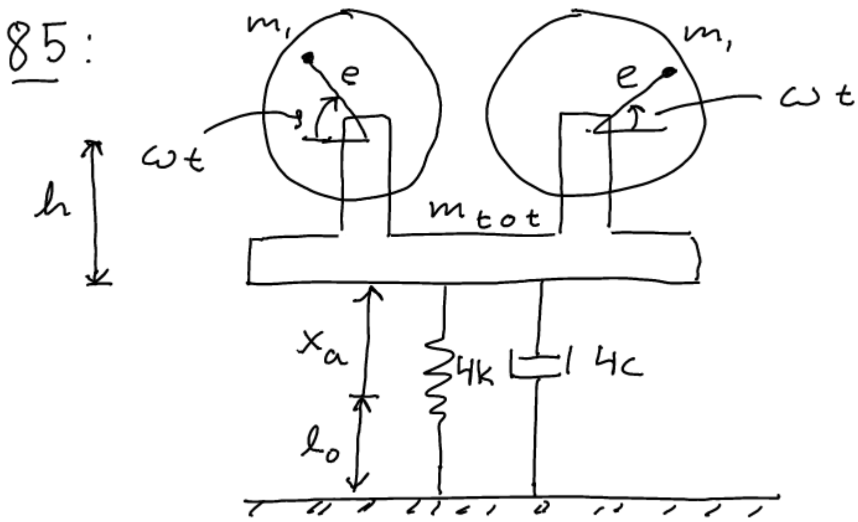
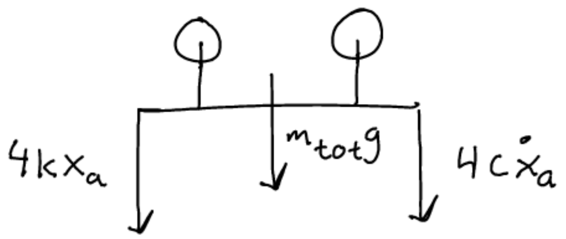


Givet:
$$k = \frac{m_{tot}\omega^2}{4}$$

Sökt: x_a i fortvarighet

Frilägg hela annordningen:



Newton II: $\left(\bar{F}^{ext} = \sum_{i=1}^{3} m_i \bar{a}_i\right)$

$$\uparrow: -m_{tot}g - 4kx_a - 4c\dot{x}_a = 2m_1(l_0 + x_a + h + e\sin\omega t)^{\cdot\cdot} +$$

$$+ (m_{tot} - 2m_1)(l_0 + x_a)^{\cdot\cdot} \Leftrightarrow$$

$$\Leftrightarrow -m_{tot}g - 4kx_a - 4c\dot{x}_a = 2m_1(\cancel{\ddot{x}_a} - e\omega^2\sin\omega t) + (m_{tot} - \cancel{2m_1})\ddot{x}_a$$

$$\Leftrightarrow \ddot{x}_a + \frac{4c}{m_{tot}}\dot{x}_a + \frac{4k}{m_{tot}}x_a = \frac{2m_1 e\omega^2}{m_{tot}}\sin(\omega t) - g \quad (1)$$

obalanskraft

$x_a = x_h + x_p$

Fortvarighet $(t \to \infty) \Rightarrow x_h = 0 \Rightarrow x_a = x_p$

Ansätt $x_p = \bar{X}_1 \cdot \sin\omega t + \bar{X}_2 \cos\omega t + C$

$\dot{x}_p = \bar{X}_1 \omega \cos t - \bar{X}_2 \omega \sin\omega t$

$\ddot{x}_p = -\bar{X}_1 \omega^2 \sin\omega t - \bar{X}_2 \omega^2 \cos\omega t$

Ins i (1) $\Rightarrow -\bar{X}_1\omega^2 \sin\omega t - \bar{X}_2 \omega^2 \cos\omega t +$

$+ \frac{4c}{m_{tot}}\bar{X}_1 \cdot \omega\cos\omega t - \frac{4c}{m_{tot}}\bar{X}_2 \omega \sin\omega t + \frac{4k}{m_{tot}}\bar{X}_1 \sin\omega t +$

$+ \frac{4k}{m_{tot}}\bar{X}_2 \cos\omega t + \frac{4k}{m_{tot}} C = \frac{2m_1 e\omega^2}{m_{tot}} \sin\omega t - g$

Identifiera:

konst: $C = \frac{-m_{tot}\, g}{4k}$

$\sin\omega t: -\bar{X}_1\omega^2 - \frac{4c}{m_{tot}}\bar{X}_2\omega + \frac{4k}{m_{tot}}\bar{X}_1 = \frac{2m_1 e\omega^2}{m_{tot}} \quad (2)$

$\cos\omega t: -\bar{X}_2\omega^2 + \frac{4c}{m_{tot}} \cdot \bar{X}_1\omega + \frac{4k}{m_{tot}}\bar{X}_2 = 0 \quad (3)$

$$(3) \Rightarrow \underline{\bar{X}}_1 = \frac{\omega^2 - \frac{4k}{m_{tot}}}{\frac{4c}{m_{tot}} \cdot \omega} \quad \underline{\bar{X}}_2 = 0$$

$$(2) \Rightarrow \underline{\bar{X}}_2 = \frac{-m_1 e \omega}{2c}$$

$$\therefore x_a = \frac{-m_1 e \omega}{2c} \cos \omega t - \frac{m_{tot} g}{4k}$$