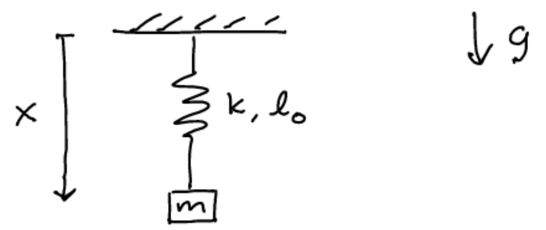
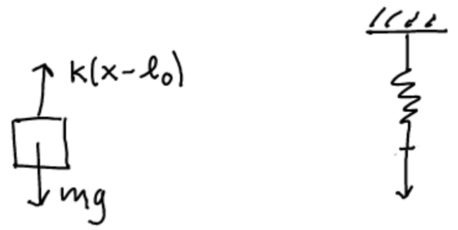
Föreläsning 7, mekanik del 1

Svängningar

Fri odämpad svängning



Frilägg:



Newton II:

$$\downarrow: \; mg - k(x-\ell_0) = m\ddot{x} \Leftrightarrow \ddot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = g + \frac{k\ell_0}{m} \qquad (1)$$

ω_n: egenvinkelfrekvens

$x = x_h + x_p$

Homogenlösning x_h:

$$\boxed{x_h = A\cos(\omega_n t) + B\sin(\omega_n t)}$$

Partikulärlösning, x_p

$x_p = C$, konst

$$(1) \Rightarrow \frac{k}{m} C = g + \frac{k l_0}{m} \Leftrightarrow C = \frac{mg}{k} + l_0$$

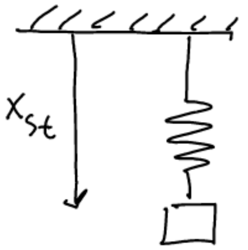
$$\therefore \; x = A\cos(\omega_n t) + B\sin(\omega_n t) + \frac{mg}{k} + l_0$$

Bestäm A och B ur givna värden på $x(0)$ och $\dot{x}(0)$

Svängningstid T_n :

$$\omega_n T_n = 2\pi \Leftrightarrow T_n = \frac{2\pi}{\omega_n}$$

Bestämning av jämviktsläge, x_{st}.



jämvikt $\ddot{x} = 0$

$$(1) \Rightarrow \frac{k}{m} x_{st} = g + \frac{k l_0}{m} \Leftrightarrow x_{st} = l_0 + \frac{mg}{k}$$

Fri dämpad svängning

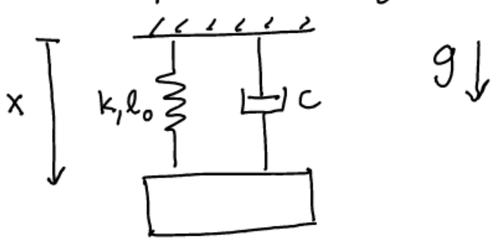
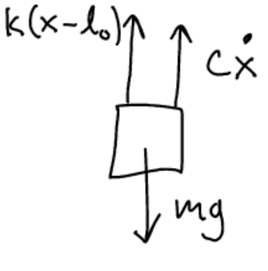


Fig 1.



Newton II:

$$\downarrow: mg - k(x - l_0) - c\dot{x} = m\ddot{x} \Leftrightarrow \ddot{x} + \underbrace{\frac{c}{m}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = g + \frac{kl_0}{m} \qquad (2)$$

grekiskt z

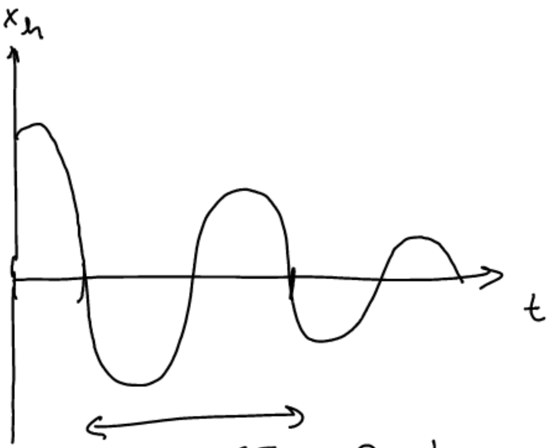
ζ dämpfaktor.

$$x = x_h + x_p$$

$\zeta < 1$: underdämpat system.

$$\boxed{\begin{aligned} x_h &= e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin(\omega_d t)) \\ \omega_d &= \sqrt{1-\zeta^2} \end{aligned}}$$

ω_d: dämpad egenvinkelfrekvens

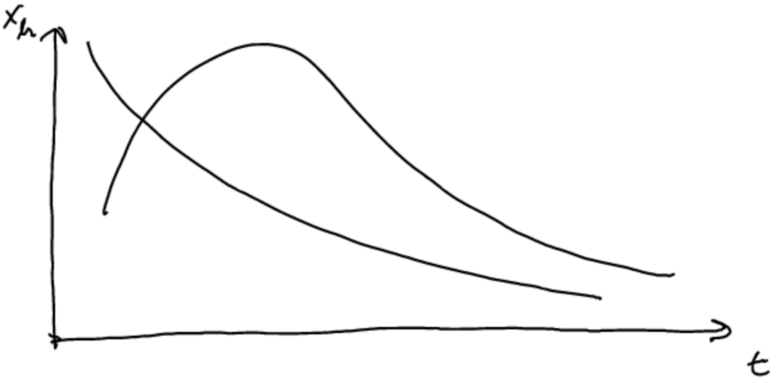


$T_d = \frac{2\pi}{\omega_d} > T_n$!

$$\boxed{\begin{array}{l} \text{Fortvarighet: } x_h \text{, dvs } x = x_p \\ \text{Lösning oberoende av begynnelsevillkoren!} \end{array}}$$

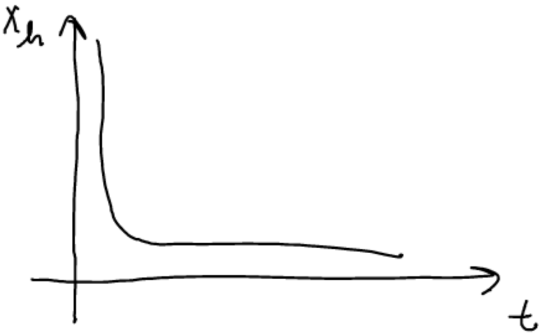
$\zeta > 1$: överdämpat system

$$x_h = A \cdot e^{-(\zeta - \sqrt{\zeta^2 - 1})\omega_n t} + B e^{-(\zeta + \sqrt{\zeta^2 - 1})\omega_n t}$$



$\zeta = 1$: Kritiskt dämpat system.

$$\boxed{x_h = (A + Bt)e^{-\omega_n t}}$$



Konvergerar snabbare
än för överdämpat!

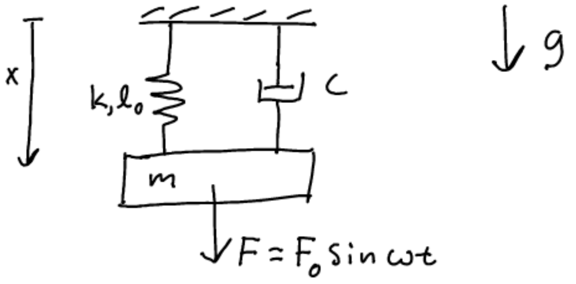
Ex $\boxed{\text{Fig 1}}$

Givet ; k, m
$\zeta = 1$

$(2) \Rightarrow 2\zeta\sqrt{\frac{k}{m}} = \frac{c}{m} \Leftrightarrow c = 2\sqrt{mk}$

Söker c.

Påtvingad, dämpad svängning



Newton II:

$\downarrow$: $mg - k(x - l_0) - c\dot{x} + F_0 \sin \omega t = m\ddot{x}$

$$\Leftrightarrow \ddot{x} + \underbrace{\frac{c}{m}}_{2\zeta\omega_n} \dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = g + \frac{k l_0}{m} + \frac{F_0}{m} \sin \omega t \quad (3)$$

$x = x_h + x_p$

vid fortvarighet: $x = x_p$ $(x_h = 0)$

$$x_p = \underline{\bar{X}}_1 + \underline{\bar{X}}_2 \sin \omega t + \underbrace{\underline{\bar{X}}_3}_{\substack{=0 \\ \text{om } \zeta = 0 \\ \text{(odämpat)}}} \cos \omega t$$

$$\dot{x}_p = \underline{\bar{X}}_2 \omega \cos \omega t - \underline{\bar{X}}_3 \omega \sin \omega t$$

$$\ddot{x}_p = -\underline{\bar{X}}_2 \omega^2 \sin \omega t - \underline{\bar{X}}_3 \omega^2 \cos \omega t$$

Ins i (3) $\Rightarrow$

$$-\underline{\bar{X}}_2 \omega^2 \sin \omega t - \underline{\bar{X}}_3 \omega^2 \cos \omega t + \frac{c}{m}\left(\underline{\bar{X}}_2 \omega \cos \omega t - \underline{\bar{X}}_3 \omega \sin \omega t\right) +$$

$$+ \frac{k}{m}\left(\underline{\bar{X}}_1 + \underline{\bar{X}}_2 \sin \omega t + \underline{\bar{X}}_3 \cos \omega t\right) =$$

$$g + \frac{k l_0}{m} + \frac{F_0}{m} \sin \omega t$$

Identifiera koefficienter:

konst: $\frac{k}{m}\underline{\bar{X}}_1 = g + \frac{k l_0}{m}$

$\sin \omega t$: $-\underline{\bar{X}}_2\omega^2 - \frac{c}{m}\underline{\bar{X}}_3\omega + \frac{k}{m}\underline{\bar{X}}_2 = \frac{F_0}{m}$

$\cos \omega t$: $-\underline{\bar{X}}_3\omega^2 + \frac{c}{m}\underline{\bar{X}}_2\omega + \frac{k}{m}\underline{\bar{X}}_3 = 0$

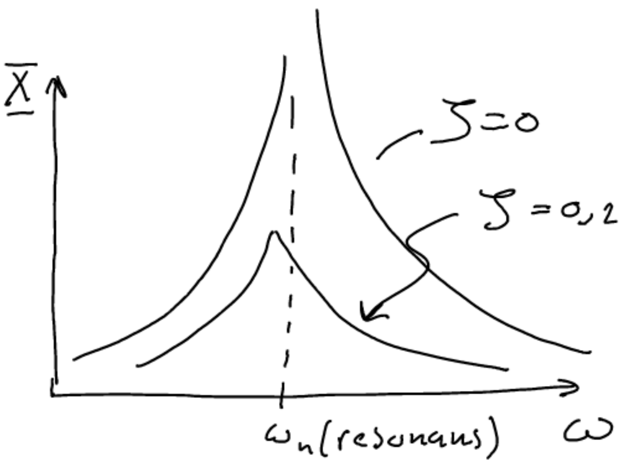
$\Rightarrow \underline{\bar{X}}_1, \underline{\bar{X}}_2, \underline{\bar{X}}_3$

kan skriva

$$x_p = \underline{\bar{x}}_1 + \underline{\bar{X}}\sin(\omega t + \varphi)$$

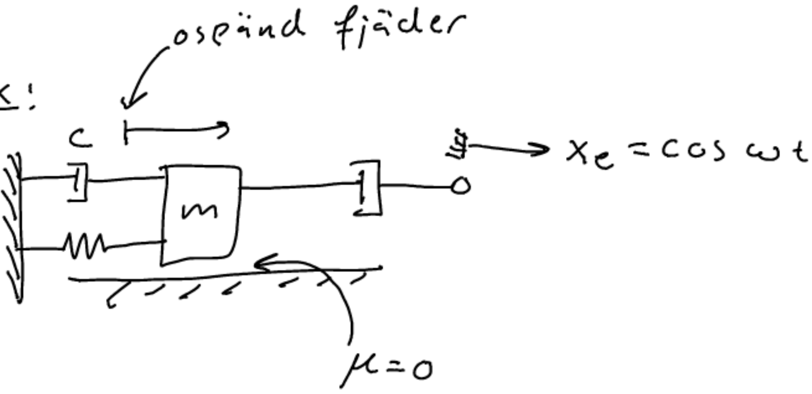
amplitud $\left(=\sqrt{\underline{\bar{x}}_2^2 + \underline{\bar{x}}_3^2}\right)$

fasvinkel



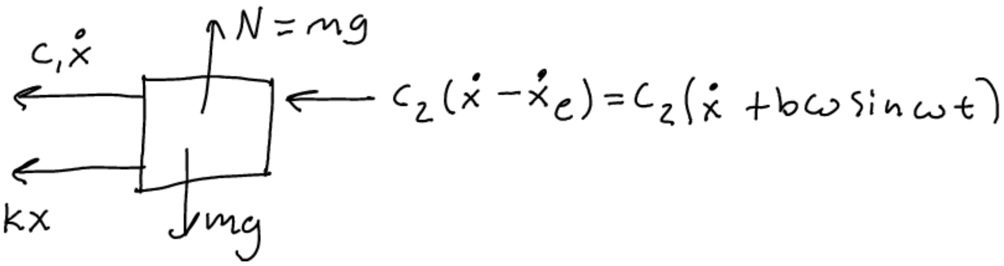
ω stor $\Rightarrow \underline{\bar{X}}$ liten.

Ex:



Diff ekv?

Frilägg:



$$\rightarrow: \quad -kx - c_1\dot{x} - c_2(\dot{x} + b\omega \sin \omega t) = m\ddot{x}$$

$$\Leftrightarrow \quad \ddot{x} + \underbrace{\frac{c_1 + c_2}{m}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2}x = -\frac{c_2 b \omega}{m}\sin \omega t$$

∎