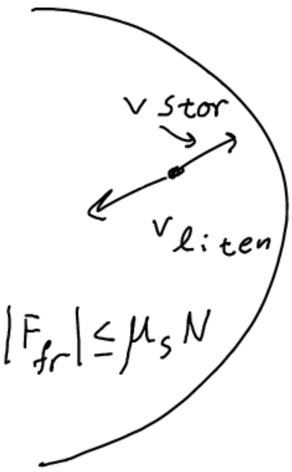
Föreläsning 4, mekanik 1.

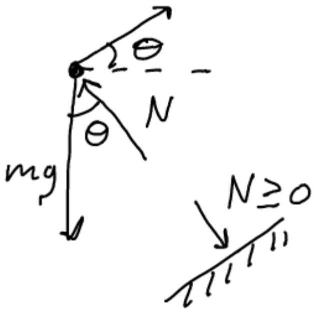
33)

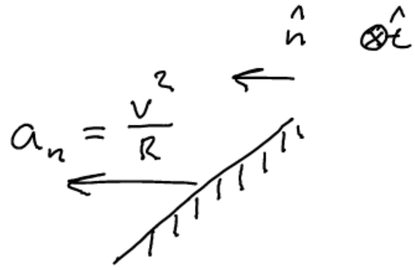
$\downarrow g$

Sökt: v så ej glider



Frilägg:



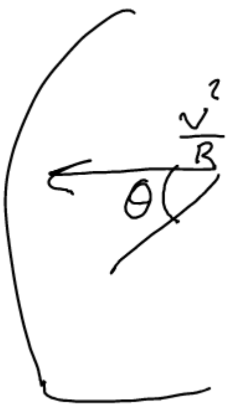


$\otimes \; a_t = \dot{v} = 0$

(cirkelrörelse)

$\nearrow$: $F_{fr} - mg\sin\theta = -\frac{mv^2}{R}\cos\theta$

$\nwarrow$: $N - mg\cos\theta = -\frac{mv^2}{R}\cos\theta$



$\therefore \; F_{fr} = mg\sin\theta - \frac{mv^2}{R}\cos\theta$

$N = mg\cos\theta + \frac{mv^2}{R}\sin\theta$

Glider ej om $|F_{fr}| \le \mu_s N$

$F_{fr} \le \mu_s N$:

$$mg\sin\theta - \frac{mv^2}{R}\cos\theta \le \mu_s mg\cos\theta + \mu_s \frac{mv^2}{R}\sin\theta \Leftrightarrow$$

$$\Leftrightarrow \frac{v^2}{Rg} \ge \frac{\sin\theta - \mu_s\cos\theta}{\mu_s\sin\theta + \cos\theta} = \frac{\tan\theta - \mu_s}{\mu_s\tan\theta + 1}$$

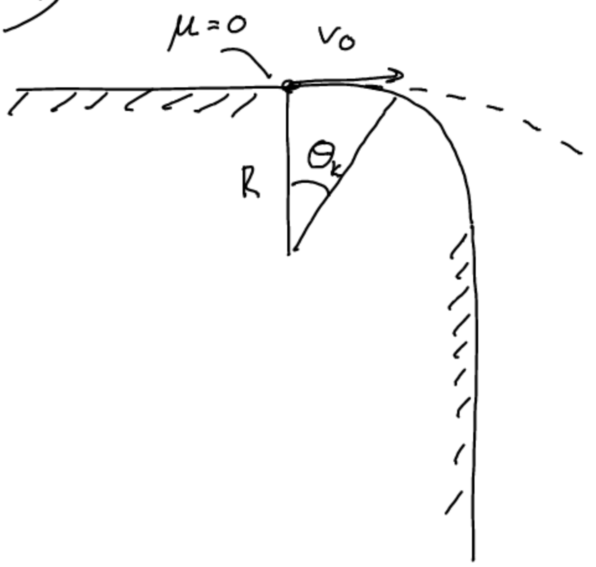
$-\mu_s N \le F_{fr}$:

$$-\mu_s mg\cos\theta - \mu_s \frac{mv^2}{R}\sin\theta \le mg\sin\theta - \frac{mv^2}{R}\cos\theta$$

$$\Leftrightarrow \frac{v^2}{gR} \le \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}$$

---

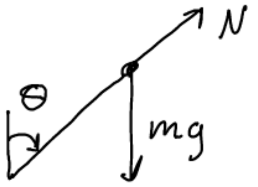
34)

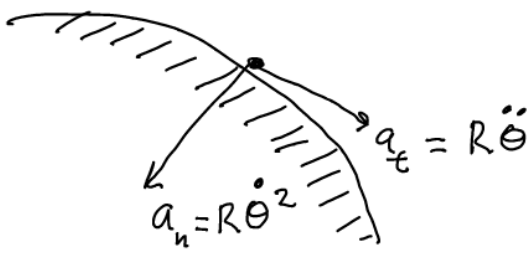


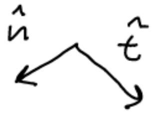
Sökt: θ_k

Teckna normalkraften N, $N = 0$ vid θ_k

Frilägg: $(\theta \le \theta_k)$







(cirkelrörelse)

Newton II:

$$\hat{n}: \quad mg\cos\theta - N = mR\dot{\theta}^2 \qquad (1)$$

$$\hat{t}: \quad mg\sin\theta = mR\ddot{\theta} \qquad (2)$$

$$(1) \Rightarrow N = mg\cos\theta - mR\dot{\theta}^2 \qquad (3)$$

$$\dot{\theta}^2 = \dot{\theta}^2(\theta)\,?$$

$$v = R\dot{\theta}$$

$$(2) \Rightarrow \ddot{\theta} = \frac{g}{R}\sin\theta$$

$$\ddot{\theta}\, d\theta = \dot{\theta}\, d\dot{\theta} \Rightarrow \int_0^{\theta} \frac{g}{R}\sin\theta\, d\theta = \int_{v_0/R}^{\dot{\theta}}$$

$$\Leftrightarrow \frac{-g}{R}\left[\cos\theta\right]_0^{\theta} = \frac{1}{2}\left[\dot{\theta}^2\right]_{v_0/R}^{\dot{\theta}}$$

$$\Leftrightarrow \frac{-g}{R}(\cos\theta - 1) = \frac{1}{2}\left(\dot{\theta}^2 - \frac{v_0^2}{R^2}\right) \Leftrightarrow$$

$$\Leftrightarrow \dot{\theta}^2 = \frac{-2g}{R}(\cos\theta - 1) + \frac{v_0^2}{R^2}$$

Insättning i 3 ⇒

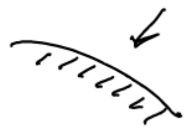
$$N = mg\cos\theta + 2mg(\cos\theta - 1) - \frac{mv_0^2}{R} \qquad (4)$$

$$N(\theta_k) = 0 \Rightarrow 3g\cos(\theta_k) - 2g - \frac{v_0^2}{R} = 0$$

$$\Rightarrow \theta_k = \arccos\left(\frac{2}{3} + \frac{v_0^2}{3Rg}\right), \quad \text{för } v_0^2 \leq gR$$

$(v_0 = 0 \Rightarrow \theta_k = 48{,}2^\circ)$

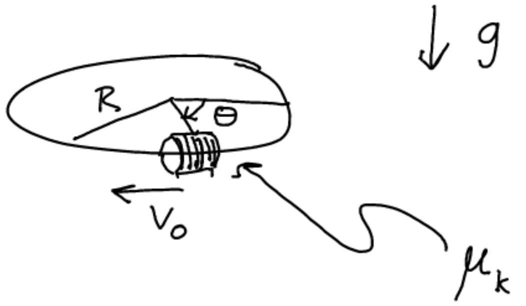
$v_0^2 > gR$: (4) ⇒ $N(\theta = 0) < 0$



$\therefore\ \theta_k = 0$ för $v_0^2 > gR$

---

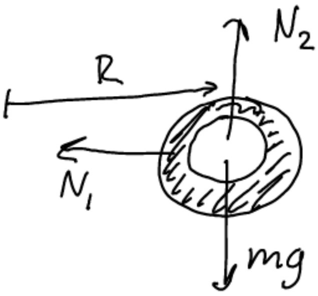
35)



Sökt: Stoppsträckan.

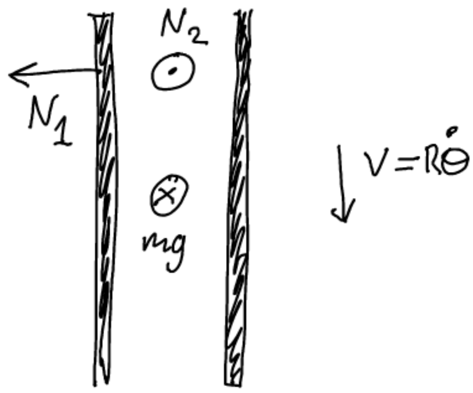
$\ddot{\theta}$?

Frilägg:



$\otimes\ F_{fr} = \mu_k\sqrt{N_1^2 + N_2^2}$

från ovan



← $a_n = R\dot{\theta}^2$

↓ $a_t = R\ddot{\theta}$

(cirkelrörelse)

Newton II

$$\hat{n}: \; N_1 = mR\dot{\theta}^2 \qquad (1)$$

$$\hat{t}: \; -\mu_k\sqrt{N_1^2+N_2^2} = mR\ddot{\theta} \qquad (2)$$

$$\hat{z}: \; mg - N_2 = 0 \qquad (3)$$

$$(1),(2) \text{ i } (3) \Rightarrow \; -\mu_k\sqrt{m^2R^2\dot{\theta}^4 + m^2g^2} = mR\ddot{\theta}$$

$$\Leftrightarrow \; \ddot{\theta} = \frac{-\mu_k}{R}\sqrt{R^2\dot{\theta}^4 + g^2}$$

Stopp-vinkeln $\longrightarrow \theta_s$

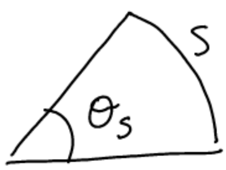
$$\ddot{\theta}\,d\theta = \dot{\theta}\,d\dot{\theta} \Rightarrow \int_0^{\theta_s} d\theta = \frac{-R}{\mu_k}\int_{v_0/R}^{0} \frac{\dot{\theta}\,d\dot{\theta}}{\sqrt{R^2\dot{\theta}^4+g^2}}$$

$$\Leftrightarrow \left[\begin{array}{ll} u = R\dot{\theta}^2 & d\dot{\theta} = \dfrac{1}{2\sqrt{uR}}\,du \\ \dot{\theta} = \sqrt{\dfrac{u}{R}} & \end{array}\right] \Leftrightarrow$$

$$\Leftrightarrow \; \theta_s = \frac{-R}{\mu_k}\int_{\frac{v_0^2}{R}}^{0} \frac{\sqrt{\frac{u}{R}}}{\sqrt{u^2+g^2}} \cdot \frac{1}{2\sqrt{uR}}\,du = \frac{-1}{2\mu_k}\int_{\frac{v_0^2}{R}}^{0} \frac{1}{\sqrt{u^2+g^2}}\,du =$$

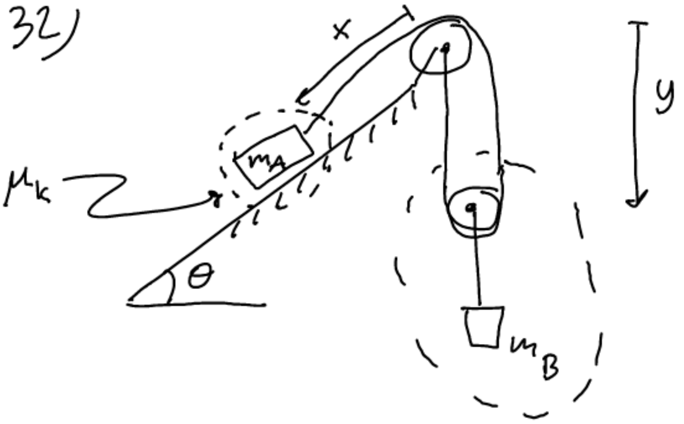
$$= \frac{-1}{2\mu_k}\left[\ln\left|u + \sqrt{u^2+g^2}\right|\right]_{\frac{v_0^2}{R}}^{0} = \frac{-1}{2\mu_k}\left(\ln g - \ln\left(\frac{v_0^2}{R} + \sqrt{\frac{v_0^4}{R^2} + g^2}\right)\right) =$$

$$= \frac{1}{2\mu_k}\ln\left(\frac{\frac{v_0^2}{R} + \sqrt{\frac{v_0^4}{R^2}+g^2}}{g}\right)$$

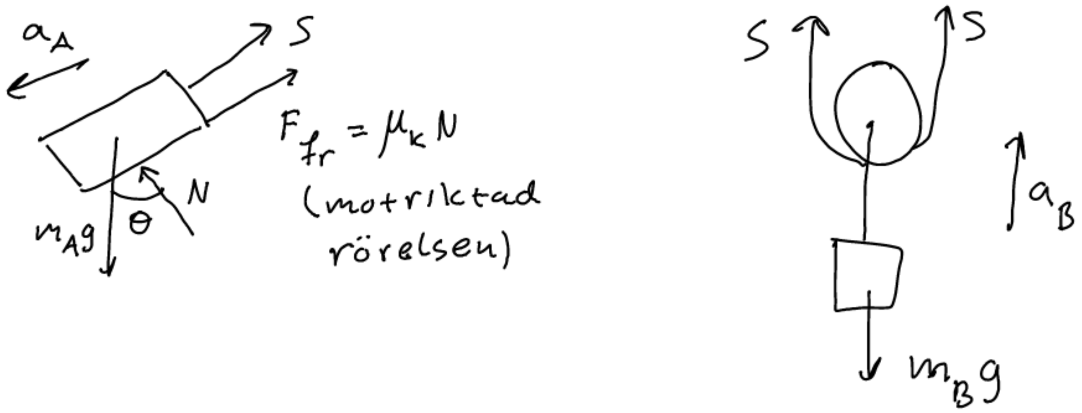


$$S = R\theta_s$$

32)



Frilägg



Newton II:

A, ↙ : $m_A g \sin\theta - \mu_k N - S = m_A a_A$ (1)

↖ : $N - m_A g \cos\theta = 0$ (2)

B, ↑: $2S - m_B g = m_B a_B$ (3)

4 obekanta, 3 ekvationer

Samband mellan a_A och a_B:

snörets längd:

$\ell = x + 2y + \text{konst.}$

ℓ konst. $\Rightarrow \dot{\ell} = \dot{x} + 2\dot{y} = 0$

$\Rightarrow \quad \ddot{\ell} = \underbrace{\ddot{x}}_{a_A} + 2\underbrace{\ddot{y}}_{-a_B} = 0$

$$\therefore \quad a_B = \frac{a_A}{2} \qquad (4).$$

$(1)-(4) \Rightarrow \ldots$