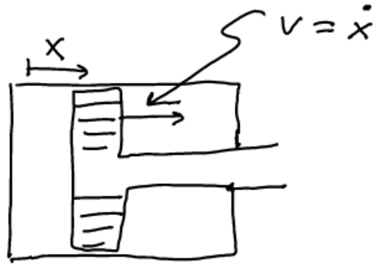
# Föreläsning 2 mekanik 1.

1) 

Givet: $t=0: x=0,\ v=v_0.$

$\dot{v}=-kv^2.$

Sökt: a) $v=v(t)$

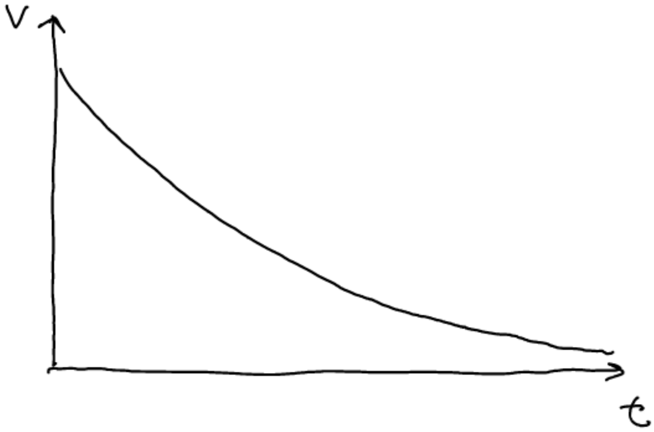
b) $x=x(t)$

c) $v=v(x).$

a)

$$\underbrace{\dot{v}}_{\frac{dv}{dt}} = -kv^2 \Rightarrow \int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t -k\,dt$$

$$\Leftrightarrow \left[\frac{-1}{v}\right]_{v_0}^{v} = -kt \Leftrightarrow -\frac{1}{v}+\frac{1}{v_0} = -kt \Leftrightarrow v = \frac{1}{\frac{1}{v_0}+kt} = \frac{v_0}{1+kv_0t}.$$



Dimensionskontroll:

$$k: \frac{m/s^2}{(m/s)^2} = \frac{1}{m}$$

$$kv_0t: \frac{1}{m}\cdot\frac{m}{s}\cdot s = 1.$$

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b) $$v=\dot{x} = \frac{v_0}{1+kv_0t} \Rightarrow x = \frac{v_0}{kv_0}\ln(1+kv_0t)+C$$

Begynnelsevillkoret (Bv):

$x(0)=0 \Rightarrow C=0$

$\therefore \quad x = \frac{1}{k}\ln(1+kv_0 t)$ .

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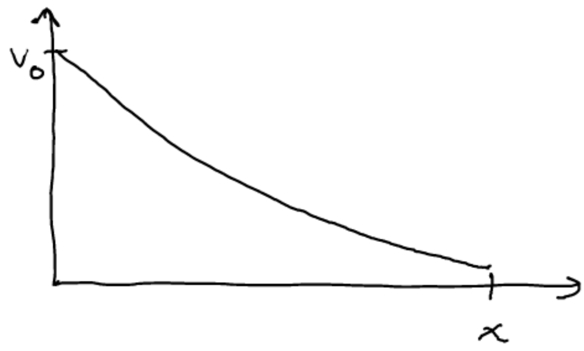
c) $\underbrace{\ddot{x}}_{\dot{v}=-kv^2} dx = \underbrace{\dot{x}\, d\dot{x}}_{v\,dv} \Rightarrow \int_0^x dx = \int_{v_0}^{v} -\frac{dv}{kv} \Leftrightarrow x = \frac{-1}{k}\ln\left(\frac{v_0}{v}\right) \Leftrightarrow$

$\Leftrightarrow \quad v = v_0 e^{-kx}$ .

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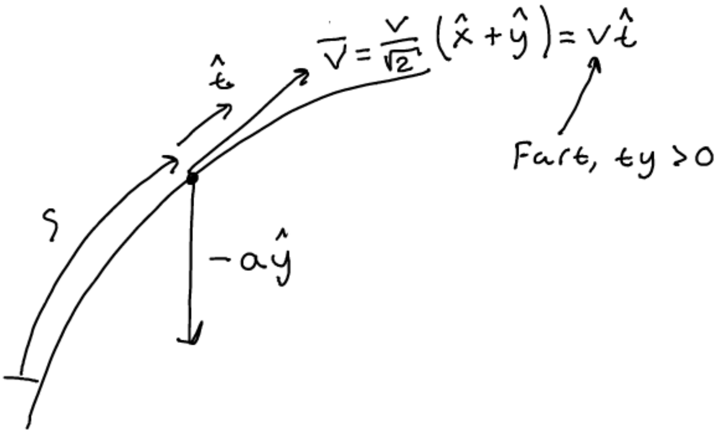
6)




Givet: $\bar{v} = \frac{v}{\sqrt{2}}(\hat{x}+\hat{y})$

$\bar{a} = -a\hat{y}$

$v, a > 0$ .

Sökt: a) $\frac{d|v|}{dt}$

b) $\hat{n}$

c) $\rho$

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a) $\bar{a} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}$ (1)

$\therefore \quad \frac{d|\bar{v}|}{dt} = \dot{v} = \bar{a}\cdot\hat{t} = -a\hat{y}\cdot \underbrace{\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})}_{\hat{t}} = \frac{-a}{\sqrt{2}}$ .

b) (1) $\Rightarrow \dfrac{v^2}{\rho}\hat{n} = \bar{a} - \dot{v}\hat{t} = -a\hat{y} + \dfrac{a}{\sqrt{2}}\cdot\dfrac{1}{\sqrt{2}}(\hat{x}+\hat{y}) =$

$$= \frac{a}{2}(\hat{x}-\hat{y}) = \underbrace{\frac{a}{\sqrt{2}}}_{\frac{v^2}{\rho}} \cdot \underbrace{\frac{1}{\sqrt{2}}(\hat{x}-\hat{y})}_{\hat{n}}$$

$\therefore \quad \hat{n} = \dfrac{1}{\sqrt{2}}(\hat{x}-\hat{y}) \qquad (\perp \hat{t}$ OK).

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c) $\dfrac{v^2}{\rho} = \dfrac{a}{\sqrt{2}} \Leftrightarrow \rho = \dfrac{\sqrt{2}}{a}v^2$.

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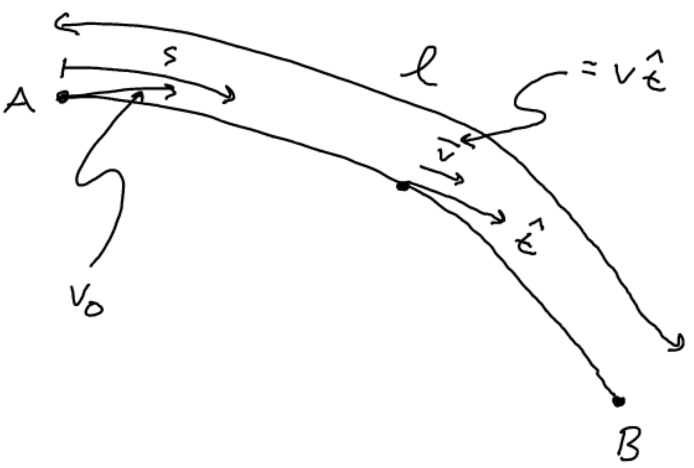


Givet: $v(s=0) = v_0$

$v(s=\ell) = 0$

$\dot{v}$ konstant

$\rho = \dfrac{k}{s}$.

Sökt: $|\bar{a}|_{max}$, motsv. s-värde.

$$\bar{a} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}$$

$$|\bar{a}| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2} \qquad (1)$$

$\dot{v}$ konstant $\Rightarrow |\bar{a}|_{max}$ då $f(s) = \dfrac{v^2}{\rho}$ är maximal. $\dot{v}$?

$$\underbrace{a_t}_{\dot{v}} ds = v\,dv \Rightarrow \dot{v}\int_0^{\ell} ds = \int_{v_0}^{0} v\,dv \Leftrightarrow \dot{v}\ell = \frac{-1}{2}v_0^2 \Leftrightarrow \dot{v} = \frac{-v_0^2}{2\ell}$$

$v = v(s)$:

$$\dot{v}\int_0^s ds = \int_{v_0}^{v} v\,dv \Leftrightarrow \dot{v}s = \frac{1}{2}\left(v^2 - v_0^2\right) \Leftrightarrow v^2 = v_0^2 + 2\dot{v}s \overset{(2)}{=} v_0^2 - \frac{v_0^2 s}{\ell}$$

$$\therefore\ f(s) = \frac{v^2}{\rho} = \frac{s}{k}\left(v_0^2 - v_0^2\frac{s}{\ell}\right).$$

$f$ maximal där $f' = 0$. $f'(s) = \frac{1}{k}\left(v_0^2 - v_0^2\frac{s}{\ell}\right) - \frac{v_0^2}{\ell}\cdot\frac{s}{k} = 0.$

$$\Leftrightarrow 1 - \frac{s}{\ell} - \frac{s}{\ell} \Leftrightarrow s = \frac{\ell}{2}.$$

$f''(s) < 0 \Rightarrow$ lokalt max.

$$(1) \Rightarrow |\bar{a}|_{max} = \ldots = \frac{v_0^2}{2}\sqrt{\frac{1}{\ell^2} + \frac{\ell^2}{4k^2}}$$

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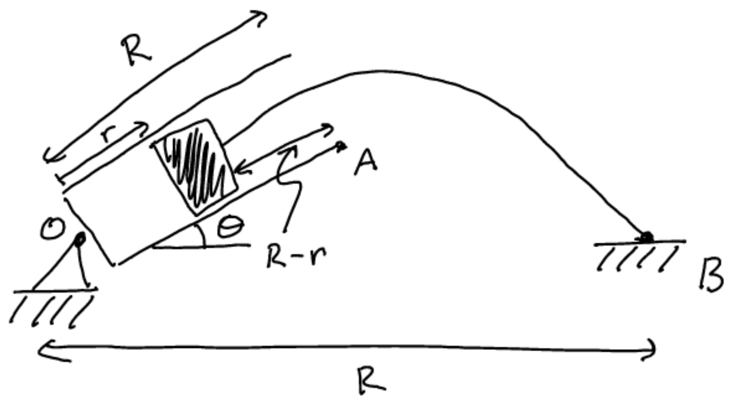


Givet $\dot{\theta} = \omega$ konst.

snörets längd $\underline{R}$.

Sökt: $|\bar{v}|$ då $\theta = 60°$

$$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$r = r(\theta)$?

snörets längd $= (R - r) + |AB| = R \Leftrightarrow r = |AB|$

cosinussatsen $\Rightarrow |AB|^2 = R^2 + R^2 - 2RR\cdot\cos\theta$

$$\therefore \quad r = \sqrt{2R^2(1-\cos\theta)} = \sqrt{2R^2 \cdot 2\sin^2\left(\frac{\theta}{2}\right)} = 2R\sin\frac{\theta}{2}$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = 2R\cos\frac{\theta}{2}\cdot\frac{1}{2}\cdot\dot{\theta} = R\omega\cos\frac{\theta}{2}$$

$$\therefore \quad |\bar{v}| = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{\left(R\omega\sqrt{3}/2\right)^2 + (R\omega)^2} = \frac{\sqrt{7}}{2}R\omega$$