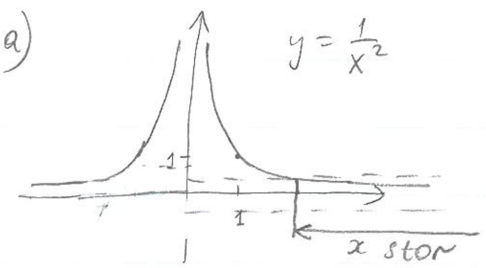
Lektion 1 (lösningar)

3.1

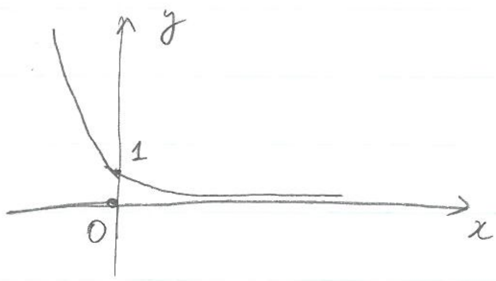
a)



$\lim_{x \to +\infty} f(x) = 0$

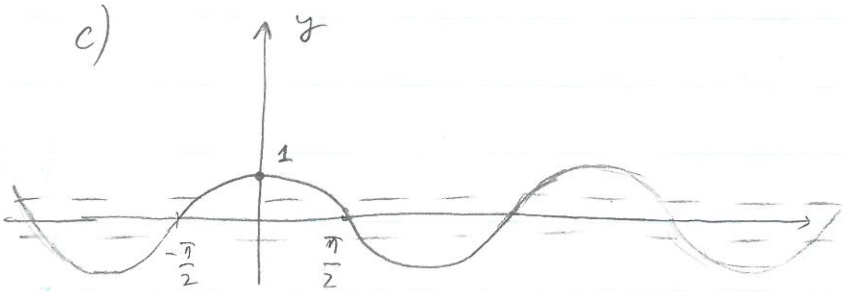
eftersom för ett "litet" band kring $y = 0$ ligger grafen i bandet för alla tillräckligt stora x.

b)



$\lim_{x \to \infty} e^{-x} = 0.$

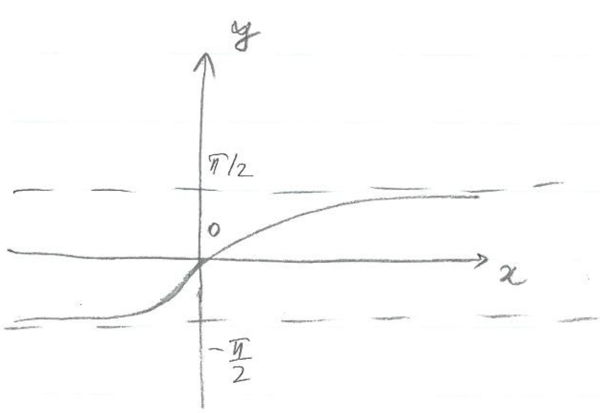
c)



$\lim_{x \to +\infty} \cos x$ (saknas / existerar inte.)

För varje litet band kommer grafen ut från banded

d)



$\lim_{x \to \infty} \arctan x = \frac{\pi}{2}$

3.2

a) $$\lim_{x\to 0} \frac{\overset{\nearrow 0}{4x^3} + \overset{\nearrow 0}{5x^2} - \overset{\nearrow 0}{7x}}{\underset{\searrow 0}{x}} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{\cancel{x}(4x^2+5x-7)}{\cancel{x}} = -7$$

b) $$\lim_{x\to\infty} \frac{x+2}{x^3+x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to\infty} \frac{x\left(1+\frac{2}{x}\right)}{x^3\left(1+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{\left(1+\overset{\nearrow 0}{\frac{2}{x}}\right)}{1+\underset{\searrow 0}{\frac{1}{x^2}}} \cdot \frac{1}{x^2} =$$

$$= \lim_{x\to\infty} \frac{1}{x^2} = 0$$

c) $$\lim_{x\to\infty} \frac{2x+\cos x}{x} = \lim_{x\to\infty} \left(2 + \frac{\cos x}{x}\right) =$$

$$= 2 + \lim_{x\to\infty} \frac{\overset{\text{Begr.}}{\cos x}}{x} = 2 + 0 = 2$$

3.7

a) $$\lim_{x\to 1} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{x-1}} =$$

$$= \lim_{x\to 1} (x+2) = 3$$

b) $$\lim_{x\to\infty} \frac{x+1}{2x-3} \cdot e^{-2x} = \left[\frac{\infty}{\infty} \overset{?}{\cdot} 0\right] =$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left(2-\frac{3}{x}\right)} \cdot \frac{1}{e^{2x}} = \frac{1}{2} \cdot 0 = 0$$

t.ex. $x \cancel{>} -1$
$<$

c) $$\lim_{x\to -1} \left(\frac{1}{x+1} + \frac{2}{x^2-1}\right) = \left[-\infty + \infty\right] =$$

$$\lim_{x\to -1} \frac{x-1+2}{(x+1)(x+1)} = \lim \frac{1}{x-1} = -\frac{1}{2}$$ $\boxed{2}$

d) $\lim\limits_{x\to\infty} \dfrac{\ln 4x}{\ln x^4} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to\infty} \dfrac{\ln 4 + \ln x}{4 \ln x} =$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{\ln x}\left(1 + \dfrac{\ln 4}{\ln x}\right)}{4\cancel{\ln x}} = \dfrac{1}{4}.$

3.10 a) $\lim\limits_{x\to 5} \dfrac{x^2 - x - 20}{x - 5} = \lim\limits_{x\to 5} \dfrac{(x+4)\cancel{(x-5)}}{\cancel{x-5}} = \boxed{9.}$

b) $\lim\limits_{x\to 1} \dfrac{x^3 - 3x^2 + 2x}{1 - x^2} = \left[\dfrac{0}{0}\right] =$

$= \lim\limits_{x\to 1} \dfrac{x(x^2 - 3x + 2)}{(1-x)(1+x)} = \lim\limits_{x\to 1} \dfrac{x\overset{-1}{\cancel{(x-1)}}(x-2)}{\cancel{(1-x)}(1+x)} =$

$= \lim\limits_{x\to 1} \dfrac{-x(x-2)}{x+1} = \dfrac{-1\cdot(-1)}{2} = \boxed{\dfrac{1}{2}}$

c) $\lim\limits_{x\to 1} \dfrac{x^4 - 1}{x^3 - 1} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{(x^2-1)(x^2+1)}{(x-1)(x^2+x+1)} =$

$= \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}(x^2+x+1)} = \boxed{\dfrac{4}{3}}$

d) $\lim\limits_{x\to 1} \dfrac{\sqrt{x} - 1}{\underset{(\sqrt{x})^2}{\underset{\parallel}{x - 1}}} = \lim\limits_{x\to 1} \dfrac{\cancel{\sqrt{x} - 1}}{\cancel{(\sqrt{x} - 1)}(\sqrt{x} + 1)} = \boxed{\dfrac{1}{2}}$

e) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2 + 5x - 1}}{3x - 7} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2\left(1 + \frac{5}{x} - \frac{1}{x^2}\right)}}{x\left(3 - \frac{7}{x}\right)} =$

$= \lim\limits_{x\to\infty} \dfrac{\overset{\text{då } x>0 \;\; x}{\overset{\parallel}{\sqrt{x^2}}}\sqrt{1 + \frac{5}{x} - \frac{1}{x^2}}}{x\left(3 - \frac{7}{x}\right)} = \boxed{1}$

f) $\lim\limits_{x\to -\infty} \dfrac{2x}{\sqrt{3x^2 + 1}} = \lim\limits_{x\to -\infty} \dfrac{2x}{\sqrt{x^2\left(3 + \frac{1}{x^2}\right)}} =$

$= \lim\limits_{x\to -\infty} \dfrac{2x}{|x|\sqrt{3 + \frac{1}{x^2}}} = \lim\limits_{x\to -\infty} \dfrac{\cancel{2x}}{\cancel{-x}\sqrt{3 + \frac{1}{x^2}}} = \boxed{-\dfrac{2}{\sqrt{3}}} \cdot \sqrt{3}$

3.8 Vi vet att $\arctan x \le f(x) \le \arctan 2x$ för alla $x > 0$.

P g a sats 3.3 (instängning) så $\lim\limits_{x\to\infty} f(x) = \frac{\pi}{2}$,

eftersom $\lim\limits_{x\to\infty} \arctan x = \lim\limits_{x\to\infty} \arctan 2x = \frac{\pi}{2}$.

På samma sätt $\lim\limits_{x\to 0} \arctan x = \lim\limits_{x\to 0} \arctan 2x = 0$,

så $\lim\limits_{x\to\infty} f(x) = 0$.

P3

3.4 a) $\lim\limits_{n\to\infty} 1 = 1$

b) $$\lim_{n\to\infty}\left(\lim_{m\to\infty} \frac{n}{n+m}\right) = \lim_{n\to\infty}\left(\lim_{m\to\infty} \frac{\overset{\text{konst}}{n}}{\underset{\text{konst}}{n} + m}\right) =$$

$$= \lim_{n\to\infty} 0 = 0$$

c) $$\lim_{m\to\infty}\left(\lim_{n\to\infty} \frac{n}{n+m}\right) = \lim_{m\to\infty}\left(\lim_{n\to\infty} \frac{n}{n(1+\frac{m}{n})}\right)\text{konst}$$

$$= \lim_{m\to\infty}\left(\lim_{n\to\infty} \frac{1}{1+\underbrace{\frac{m}{n}}_{\to 0}}\right) = \lim_{m\to\infty} 1 = 1.$$